The Traveling Salesman of the Brooklyn Subway System

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The Big Apple

- Population as of July 1, 1999: 7,428,162
- Land Area: 302 square miles
- People per Square Mile: Approximately 24,600
Public Transportation

- Taxi Cab
- Bus
- Subway

http://perso.wanadoo.fr/cfm/nyc/#TaxisOn5th
http://www.mta.nyc.ny.us/index.html
Infatuation of the Subway

- Over 5.1 million ride everyday
- 1.3 billion ride in a year
- Uses enough power to light the city of Buffalo for a year
- 9,820 automatic train stops
- 1 million miles per day
- 25 routes operating 24 hours a day
- 842 miles of track
Subway History

- 1863 - World’s first subway in London
- 1870 - First unofficial subway in New York
- October 27, 1904 - NYC official subway
Traveling Salesman Problem on the Subway

Suppose a salesman is traveling by the subway and he must stop at select stations in a certain area. What is the sequence of the stops so that the total time on the subway is a minimum?
History of T.S.P

- 1759 – Euler published a solution to the Knight’s Tour Problem in chess
- 1934 – H. Whitney posed the problem as a T.S.P.
- 1954 – Provably optimal tour found of 49 cities
- 1980 – Provably optimal solution to a 318-city problem
Approximate Algorithms

- Nearest Neighbor Algorithm
- Nearest Insertion Method
- Farthest Insertion Method
- 2-opt Improvement Method
Nearest Neighbor Algorithm

- Pick any starting node
- Go to the nearest node not yet visited
- Continue from there to the nearest unvisited node
- Repeat this until all points have been visited
- Then return to the starting point.
Nearest Insertion Method

- Start with two nodes that have a high-time tour.
- For each uninserted node, figure the minimum time between it and an inserted node.
- Insert the node that has the minimum time.
Nearest Insertion

- Time:
  - 2
  - 3
  - 4
  - 5
  - 6
  - 7
  - 8
  - 9
  - 10
  - 11
  - 12
  - 13
  - 14
  - 15
  - 16
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  - 31
  - 32
  - 33
  - 34
  - 35
  - 36
  - 37
  - 38
  - 39
  - 40

- Locations:
  - McDowell
  - Coney Island
Farthest Insertion Method

- Start with two nodes that have a high-time tour.
- For each uninserted node, figure the minimum time between it and an inserted node.
- Insert the node that has the maximum time.
Farthest Insertion

Coney Island

McDowell 24
Farthest Insertion

Time
17
Farthest Insertion
Farthest Insertion

Coney Island

McDowell 27

Time
17
23
32
16
15
12
13
10
11
4
14
167
2-Opt Improvement Method

- Consider all edges of a tour.
- Remove two edges between a total of four nodes.
- Connect the start of one deleted edge to the start of the other deleted edge.
- Connect the end of one deleted edge to the end of the other deleted edge.
2-Opt

Coney Island

McDowell 30

Time
K-Opt Improvement Method

- Consider all edges of a tour.
- Remove k edges between select nodes.
- Connect the start of one deleted edge to the start of the other deleted edge.
- Connect the end of one deleted edge to the end of the other deleted edge.
Best Solution of Chosen Algorithms

- Nearest Neighbor = 126 minutes
- Nearest Insertion = 110 minutes
- Farthest Insertion = 167 minutes
- 2-Opt Improvement = 126 minutes

Note: This will not always be the case.
Conclusion

- There is no proven efficient algorithm for the Traveling Salesman Problem. There are algorithms available, but not one always produces the minimum time. The algorithms can give insight to problems and how we relate to them in the real world.