FRAC TALS

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Senior Seminar Project
Chaos

- Chaos is apparently unpredictable behavior arising in a deterministic system because of great sensitivity to initial conditions.
- The idea that seemingly chaotic and disconnected systems actually do have order.
“Can the flap of a butterfly’s wing stir up a tornado in Texas?”

-Edward Lorenz
Role of the mathematician

- This idea intrigued scientists and mathematicians.
- If they could find some pattern, then maybe they could predict things such as weather.
What is in common?
What in the world is a Fractal?

Named by Mandelbrot in 1975, from Latin *fractus* meaning “to break”.

- A fractal is a geometrical figure in which an identical motif repeats itself on an ever diminishing scale.
- A fractal is an image that repeats itself over and over again within itself.
What do fractals have to do with chaos?

Fractals have some of the same chaotic characteristics.

Fractals are:

- Sensitive to small changes
- Unpredictable
- Appear chaotic, even though they were created using non-chaotic equations.
Key Ideas

- Recursion
- Iterations
- Self-Similarity
- Fractional Dimension
Why in Mathematics?

- Mathematical equations can be assigned to explain the recurring nature of the fractals.
Fractals made simple

Key concepts:

- Functions
- Graphing
- Complex numbers
Background Information

- Fractals use complex numbers instead of the familiar (x,y) coordinates.
- Use $x+iy$ instead
  - $X = $ real
  - $Y = $ imaginary
- Use a different coordinate plane
  - $X$-axis = real numbers
  - $Y$-axis = imaginary numbers
The Mandelbrot

The Julia Set

http://spanky.triumf.ca/www/fractint/julia_type.html
Using Computers to Generate

Using the equation:  \( z_n^2 = z_{n-1}^2 + c \)

- Pick a point:  \( 2+1i \)
- Pick a constant: \( c=0 \)
- Substitute in the equation:
  \[
  z_n = z_{2+i} = (2+i)(2+i) + 0 = 4+2i+2i+i^2
  \]
  \[= 3+4i \]
Recursion

- Now that we have a new value we will execute the function with this value. This idea is called recursion. This can also be called iterating the problem. The more iterations, the more complex.

- Remember to keep the constant the same for each different Julia Set.
Recursion in Action

With the same formula as before:

\[ z_n^2 = z_{n-1}^2 + c \]

Substitute our new value of 3+4i

\[ z_n^2 = (3+4i)^2 + 0 \]
\[ = (3+4i)(3+4i)+0 \]
\[ = 9+12i+12i+16i^2 \]
\[ = -7+24i \]
Julia Set for c=0

Courtesy of Geometer's Sketchpad
Key terms in the complex plane

- **Escape Set** – points for which the iteration produces values that are unbounded.
- **Prisoner Set** – points for which the iteration produces values that are bounded.
- **Boundary** – points for which every neighborhood contains points from both the escape and prisoner sets.
- Escape set
- Prisoner Set
- Boundary Set
What's next?

- All the points created with a constant form a Julia Set.
- The Julia Sets that are connected are in the Mandelbrot set, those disconnected are not.
Definitions

- A set is called **connected** provided it cannot be decomposed into two disjoint, non-empty sets.

- A set is called **disconnected** if it can be decomposed into disjoint parts.
Examples

Connected

Disconnected

http://www.cs.unca.edu/~mcclure/cgi-bin/julia.cgi?c=-1.298+-0.6413333333333331i
Now what?

- The areas that are dark blue are the corresponding connected Julia sets.
- All other areas are the corresponding disconnected Julia Sets.
Are the Colors significant?

YES!

The colors tell whether the point is in the set or not.

The different colors are symbolic of the different number of iterations.
The Koch Curve

Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html
The Koch Curve

http://emmy.dartmouth.edu/~c18w99/webpages/nkk/koch.html
Sierpinski’s Triangle

Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html
Fractals in the “Real World”

- Human Body
- Nature
- Food
- Landscapes
- Coastlines
- Stock Market
- Weather
- Etc.
The Human Body

- Neurons
- Cells
- Pathogens
- Brain
- Blood vessels
Capillaries

http://spanky.triumf.ca/www/other_images.html
Ferns and other Plants

http://www.mste.uiuc.edu/courses/ci330ms/eilken/unitnatu.HTML
Trees

- Branches
- Leaves
Cauliflower

http://cosmopolis.com/dl/fractals-exhibit.html
Lightning

http://www.redflagsdaily.com/harmony.html
Fractal-generated clouds

http://www.kcsd.k12.pa.us/~projects/fractal/pics.html
Mountain Range

Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html
Landscape wrapped around a sphere (computer generated)
Snowflakes

Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html
Conclusion

- Fractals are all around us.
- Mathematicians have developed and are continuing to develop equations to generate these fractals.
- Maybe the stock market and weather will be even more predictable in the future.