Substitution Groups and Remarkable Subgroups:

Using Galois' Definitions in the Pedagogy of Elementary Group Theory

Pew Research Grant Proposal - Fall 2005

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"*it seems to be one of the laws of mathematical history that if a concept can be detached from its origins, it will be*."- John Stillwell in Elements of Algebra

I. Description of the project and its major goals.

Mathematics is a creative process and unfortunately that process is often hidden from students of the discipline. This is certainly the case in the area of mathematics commonly referred to as abstract algebra. Current pedagogy conceals from the student many of the great ideas generated by significant problems in the history of the discipline. B. Melvin Kiernan asserts that "without a clear historical perspective it is difficult to see or even imagine the connection between the [abstract] algebra of the present day and the computational problems from which it arose." Therefore, it seems reasonable to suggest that mathematics professors, by incorporating historical perspective and original ideas into their pedagogy, could enhance the learning process for their students.

This summer, I propose to build upon my previous study of the mathematical ideas of Evariste Galois (Pew Summer Research Grant 2001). The previously funded research produced a chapter entitled "Using Galois' Ideas in the Teaching of Abstract Algebra" in the forthcoming Mathematical Association of America's Note Series, From Calculus to Computers: Using the Last 200 Years of Mathematical History in the Classroom. The chapter gives an outline for structuring a first course in abstract algebra using the ideas of Galois. Now, I want to extend this research by exploring how Galois' original definitions of three key concepts: *group* (as a group of substitutions or permutations), *normal subgroup* (as a smaller group with a "remarkable" property), and

solvable (a group that can be successively diminished until it contains only one permutation), can be used to enhance the learning of students as they encounter these concepts in a first course in abstract algebra.

I will focus my research on Galois' famous paper *Memoirs on the Conditions for Solvability of Equations by Radicals*. The publication of this memoir in 1846 along with Galois' other mathematical works marks the beginning of Galois Theory, an area of mathematical research that has exerted a tremendous influence on the development of abstract algebra. Galois was the first mathematician to introduce the mathematical term *group*. Furthermore, Galois identified a specific property of groups, now known as *solvability*, which enabled him to translate his original problem from the theory of equations into an equivalent problem within the newly established theory of groups. The solvability of a group is determined by examining the *subgroup* structure of the group. The 20th century algebraist I. N. Herstein said, "It is a tribute to the genius of Galois that he recognized that those subgroups for which the left and right cosets coincide are distinguished ones. Very often in mathematics the crucial problem is to recognize and to discover what are the relevant concepts; once this is accomplished the job may be more than half done." Today, these "remarkable" subgroups are known as *normal subgroups*. The notions of *group*, *normal subgroup*, and *solvable group* are essential concepts in the study of elementary group theory.

Upon completion of my study of this memoir, I will be able to answer the following question: How can Galois' original definitions of *group*, *normal subgroup*, and *solvable group*, be incorporated into the pedagogy of elementary group theory within a first course in abstract algebra? I will write an article, which I will present at a national mathematics meeting and subsequently submit for publication, elucidating these definitions of Galois and indicating how an instructor of abstract algebra can use these definitions to augment the teaching of elementary group theory and the learning of the student.

II. Review of Scholarly Literature

Although much has been written about the brief life of Evariste Galois and about the branch of abstract algebra known as Galois Theory, very little has been written about how one might use Galois' original definitions of *group*, *normal subgroup*, and *solvable group* as motivation for the study of elementary group theory. Thus, to my knowledge, the research project I have outlined is novel. It is an attempt to integrate the scholarship that exists on the historical development of Galois Theory with the pedagogical scholarship that calls for the motivation of mathematical ideas through the use of history and original sources.

The historical evolution of Galois theory from its beginnings up to its formulation by Emil Artin in the late 1930's can be found in B. Melvin Keirnan's *The Development of Galois Theory from Lagrange to Artin* [1971] and B. L. van der Waerden's *Die Galois-Theorie von Heinrich Weber bis Emil Artin* [1972]. In particular, Keirnan discusses the origins of Galois' ideas in the works of Lagrange, Ruffini, Gauss, Abel, and Cauchy. Two books, Galois Theory [1984] by H. M. Edwards and Galois' Theory of Algebraic Equations [1988] by J. P. Tignol examine Galois Theory from an early 19th century perspective. The Edwards book includes the only published English translation of *Memoirs on the Conditions for Solvability of Equations by Radicals.* The influence of Galois' ideas is addressed in many books on abstract algebra including: Elements of Algebra [1994] and Mathematics and Its History [1989] by John Stillwell, Galois Theory [1990] by Joseph Rotman, and A History of Algebra [1985] by B. L. van der Waerden. A recent comprehensive biography Evariste Galois (1811-1832) [1996] has been written by Laura Toti Rigatelli.

There is growing amount of scholarship on the use of the history of mathematics and original sources in the teaching of the discipline. I was first introduced to the concept of teaching with original sources by Laubenbacher, Pengelley, and Siddoway in their paper *Recovering Motivation in Mathematics: Teaching with Original Sources* [1994]. Laubenbacher and Pengelley have written several other articles including *Mathematical Masterpieces: Teaching with Original Sources* [1996]. The Mathematical Association of

America (MAA) has published several texts: Learn From the Masters [1995], Vita Mathematica: Historical Research and Integration with Teaching [1996], Using History to Teach Mathematics [2000], and From Calculus to Computers: Using the Last 200 Years of Mathematical History in the Classroom [forthcoming, 2005] which contain significant historical ideas and insights to incorporate into the undergraduate curriculum. In addition to these articles and texts, several source books have been recently published including Classics of Mathematics [1995] by Calinger. Israel Kleiner of York University has written an article A Historically Focused Course in Abstract Algebra [1998] describing a course for future secondary teachers of mathematics using a problem-based historical approach.

III. Time Frame for Completion and Dissemination of the Project

Spring 2006

- Identify sources that need to be read or reviewed.
- Plan trip and itinerary.

June-July 2006

- Travel to Louisville. I plan to visit the Bullitt Rare Books Collection at the University of Louisville. The Collection includes an original version of Liouville's 1846 publication of Galois' work with introduction.
- Travel to Knoxville. Present a preliminary report on the results of my research at Mathfest 2006 (the annual Summer meeting of the Mathematical Association of America).

August 2006

- Write draft of article. Search for appropriate venue to disseminate article.

Fall 2006

- Finish article and submit for review and publication.

Spring 2007

- Report to Pew Selection Committee.

IV. Budget

Travel (Louisville)	$240
Lodging (Louisville)	400
Food	160
Misc expenses	100
Salary (= pay for two summer courses)	3600
Total	**$4500**

V. Essay on the integration of faith and discipline

The integration of faith and learning naturally leads to an examination of the historical and philosophical foundations of the discipline. The Christian scholar should explore both the historical events and the significant ideas that have shaped the discipline. The Christian professor should ask *why* certain disciplinary areas of knowledge are selected for inclusion in the curriculum and *how* are these specific areas presented to students. The research project outlined here investigates the historical foundations of one area of mathematics, namely abstract algebra, and in particular, the original definitions of *group*, *normal subgroup*, and *solvable*. The goal is simply the recovery of perspective and motivation in the presentation of these concepts in a first course that discusses elementary group theory.

Mathematics is a human endeavor. The ability to *do* mathematics is a gift from God. Albert Einstein marveled that, "the only incomprehensible thing about the universe is that it is comprehensible." Remarkably, God has privileged humanity with the capability to explore His creation through the use of reason and rational thought and to draw necessary conclusions from His design. Mathematics is the intellectual activity that often best anticipates and expresses the outcomes of this exploration; it is, perhaps, the principal instrument through which understanding is achieved. Therefore, I consider it my responsibility not only to *do* mathematics but also to *teach* mathematics. It is a privilege to share my discipline and to make it more enjoyable and accessible to students and the greater community.