The Traveling Salesman of the Brooklyn Subway System

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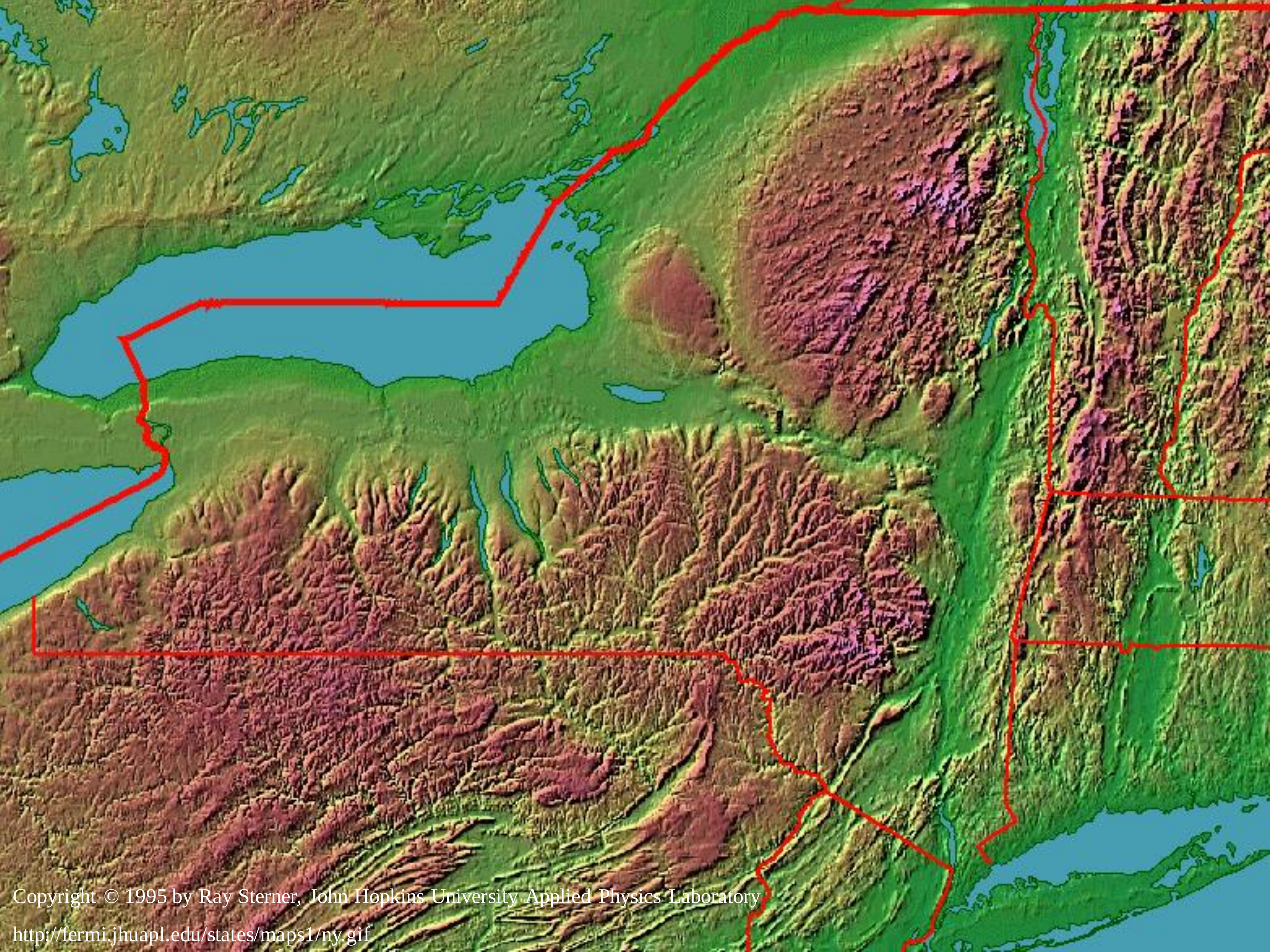

http://compuserve.digitalcity.com/newyork/vacationpics/main.dci?plAction=display&plSkip=7#photolib


http://antwrp.gsfc.nasa.gov/apod/ap970408.html

The Big Apple

- Population as of July 1, 1999
  7,428,162
- Land Area
  302 square miles
- People per Square Mile
  Approximately 24,600

Public Transportation

- Taxi Cab
- Bus
- Subway

http://perso.wanadoo.fr/cfm/nyc/#TaxisOn5th

http://www.mta.nyc.ny.us/index.html

Infatuation of the Subway

- Over 5.1 million ride everyday
- 1.3 billion ride in a year
- Uses enough power to light the city of Buffalo for a year
- 9,820 automatic train stops
- 1 million miles per day
- 25 routes operating 24 hours a day
- 842 miles of track

Subway History

- 1863 - World's first subway in London
- 1870 - First unofficial subway in New York
- October 27, 1904 - NYC official subway

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Subway Map

http://www.mta.nyc.ny.us/nyct/maps/submap.htm

- Manhattan
- Bronx
- Queens
- Brooklyn
- Harlem

Traveling Salesman Problem on the Subway

- **Suppose a salesman is traveling by the subway and he must stop at select stations in a certain area. What is the sequence of the stops so that the total time on the subway is a minimum?**

History of T.S.P

- 1759 – Euler published a solution to the Knight’s Tour Problem in chess
- 1934 – H. Whitney posed the problem as a T.S.P.
- 1954 – Provably optimal tour found of 49 cities
- 1980 – Provably optimal solution to a 318-city problem

BROOKLYN

Saratoga Av
Sutter Av–Rutland Rd
Crown Hts Utica Av
Kingston Av
Nostrand Av
President St
Sterling St
Winthrop St
Church Av
Beverly Rd
Newkirk Av
Flatbush Av Brooklyn College
Kingston-Throop Avs
Nostrand Av
Franklin Av
Clinton-Washington
Jay St
Hoyt St
DeKalb Av
Fulton St
Lafayette Av
Atlantic Av
Park Pl
7 Av
Botanic Garden
Prospect Park
PROSPECT PARK
Parkside Av
Bergen St
Carroll St
Smith-9 Sts
4 Av 9 St
Union St
Pacific St
Bergen St
Grand Army Plaza
Eastern Pkwy Brooklyn Museum
7 Av
15 St Prospect Park
Prospect Av
25 St
36 St
45 St
53 St
59 St
Bay Ridge Av
77 St
86 St
Bay Ridge 95 St
8 Av
Fort Hamilton Pkwy
New Utrecht Av
9 Av
Fort Hamilton Pkwy
50 St
55 St
62 St
71 St
79 St
18 Av
20 Av
Bay Pkwy
25 Av
Bay 50 St
Fort Hamilton Pkwy
Church Av
Ditmas Av
18 Av
Av I
Bay Pkwy
Av N
Av P
Kings Hwy
Av U
Av X
Neptune Av
18 Av
20 Av
Bay Pkwy
Kings Hwy
Avenue U
86 St
Church Av
Beverley Rd
Cortelyou Rd
Newkirk Av
Avenue H
Avenue J
Avenue M
Kings Highway
Avenue U
Neck Rd
Sheepshead Bay
Brighton Beach
Ocean Pkwy
West 8 St–NY Aquarium
Coney Island Stillwell Av (B only)
BROOKLYN

Saratoga Av
East 105 St
Sutter Av–Rutland Rd
Crown Hts
Utica Av
Canarsie
Rockaway Pkwy
Kingston Av
Kingston-Throop Avs
Nostrand Av
Franklin Av
Clinton-Washington Avs
Lafayette Av
Fulton St
DeKalb Av
Hoyt St
Atlantic Av
Park Pl
7 Av
Nostrand Av
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Sterling St
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86 St
Bay Ridge
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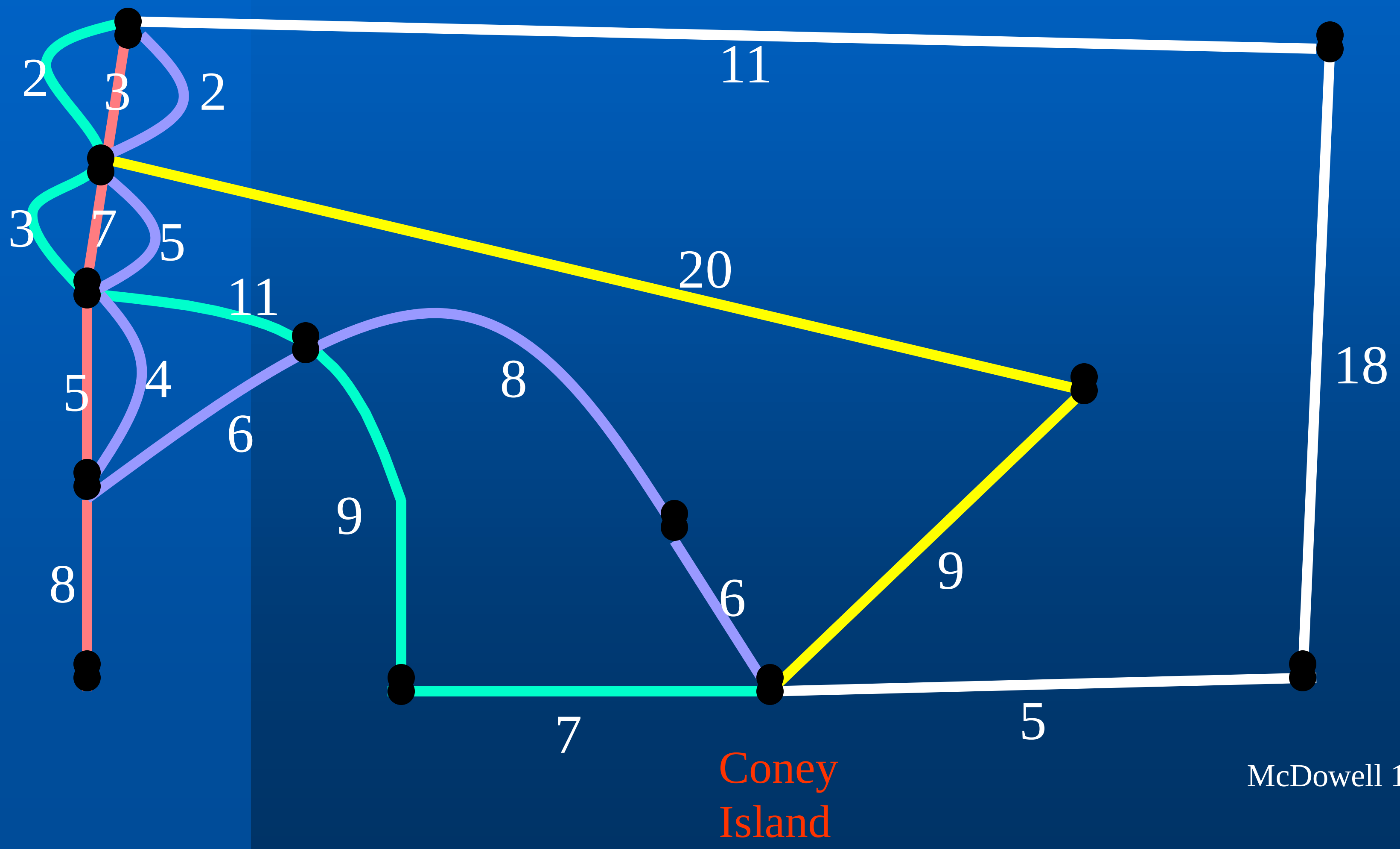
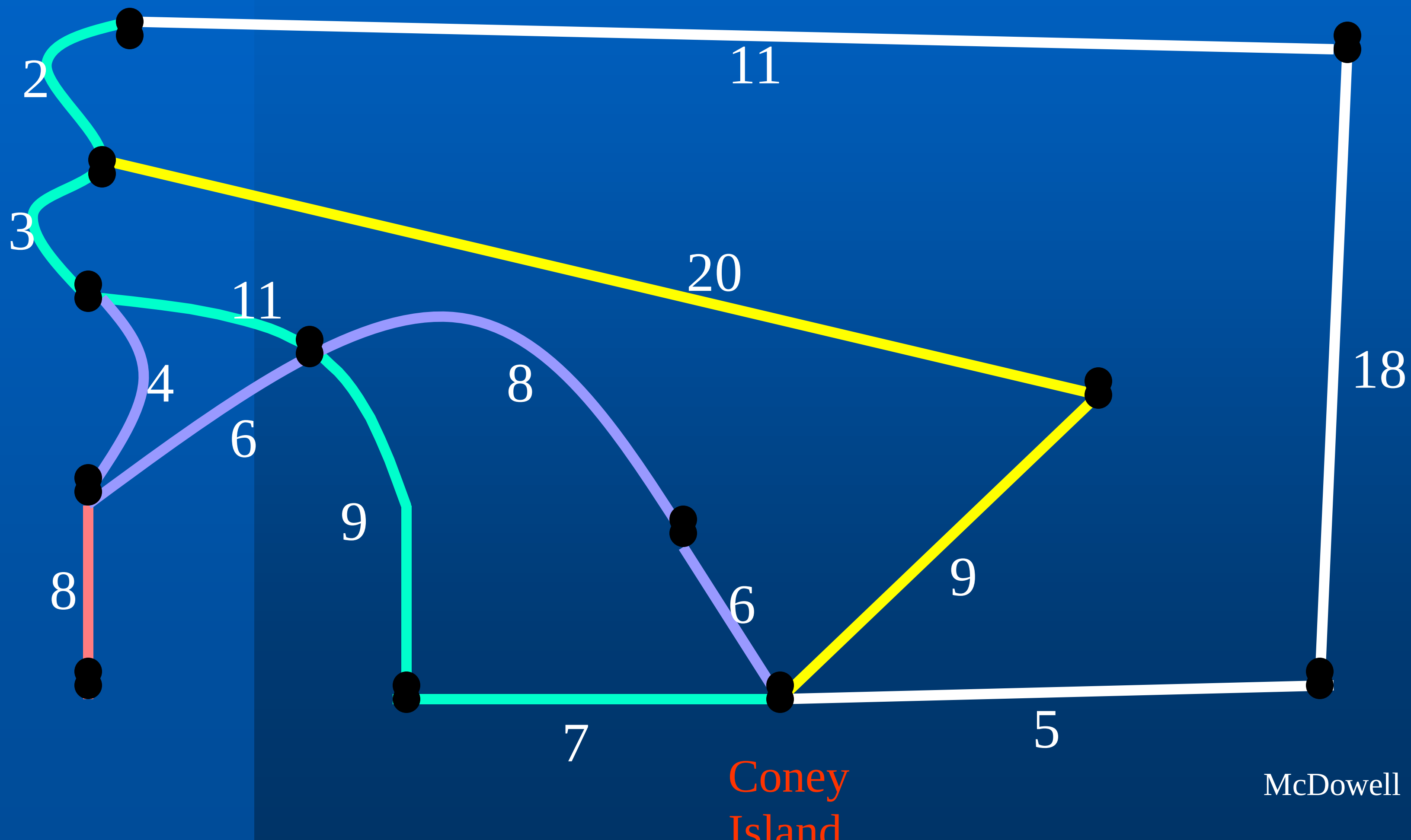
Ditmas Av
18 Av
Av I
Bay Pkwy
Av N
Av P
Kings Hwy
Av U
Av X
Neptune Av
8 Av
Fort Hamilton Pkwy
18 Av
20 Av
Bay Pkwy
Kings Hwy
Avenue U
86 St
BROOKLYN
Jamaica Bay
Broad Channel
Beach 90 St
Beach 98 St
Beach 105 St
Rockaway Park
Beach 116 St

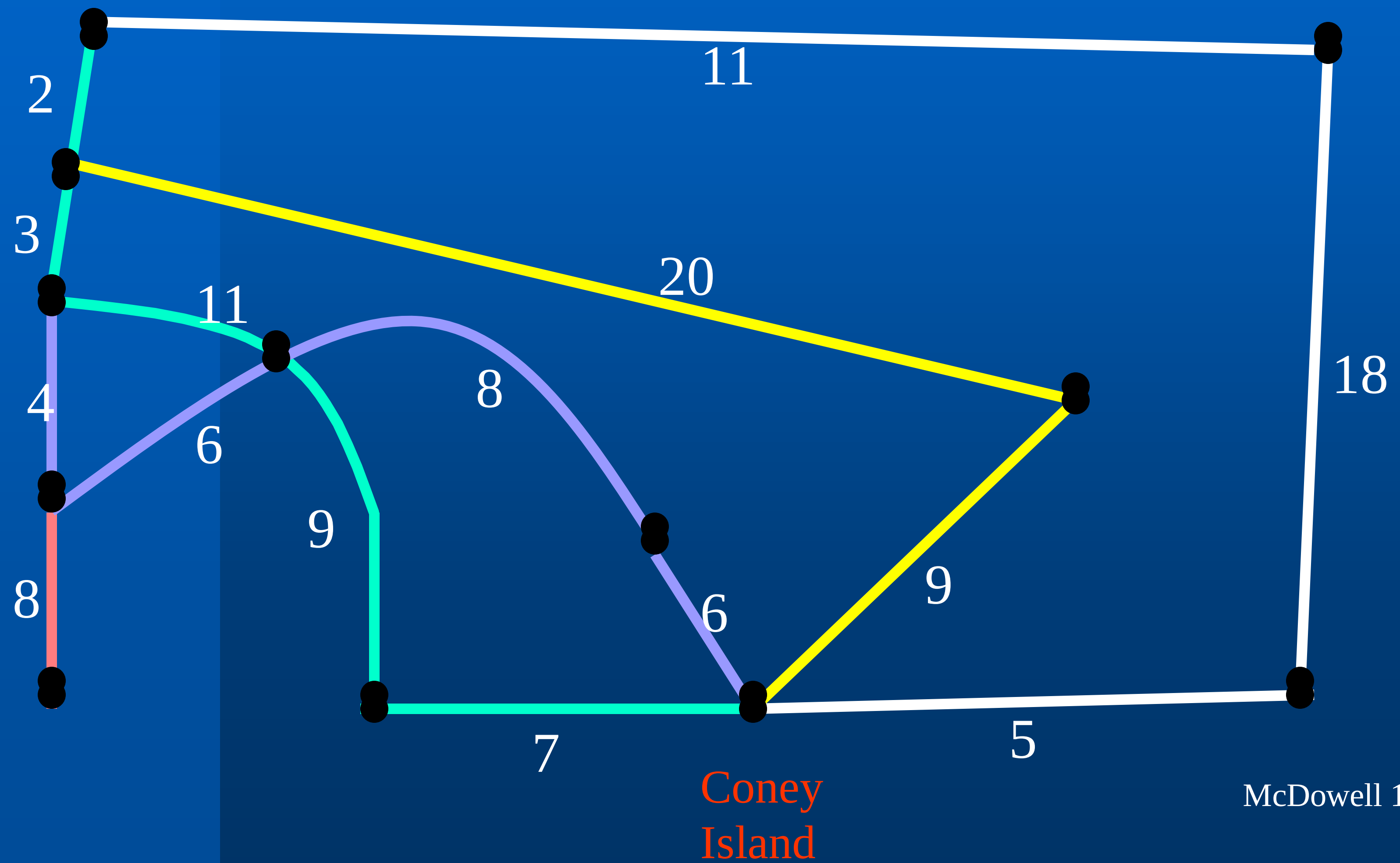
Subway Map



Subway Map



Subway Map



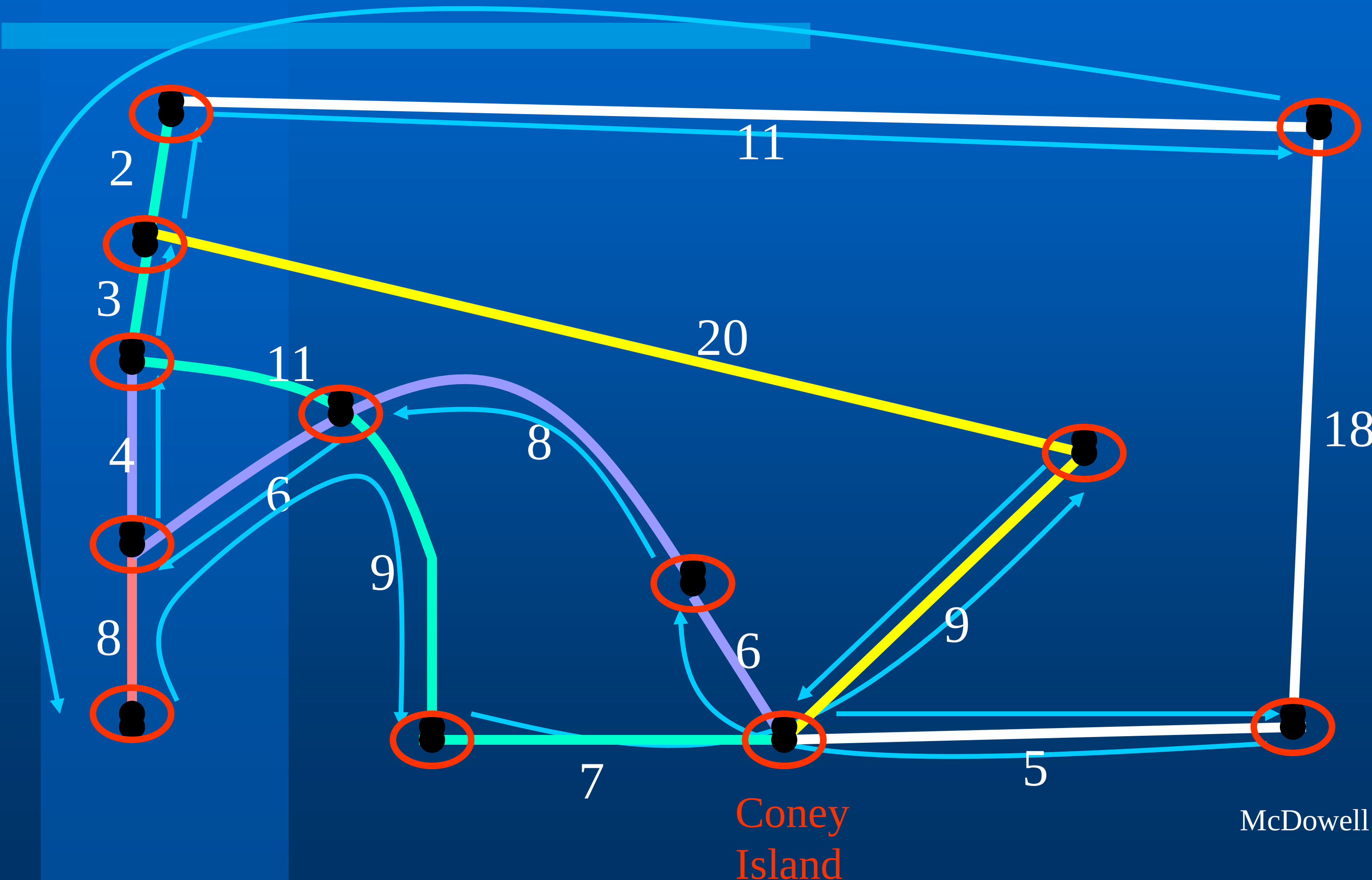
Approximate Algorithms

- Nearest Neighbor Algorithm
- Nearest Insertion Method
- Farthest Insertion Method
- 2-opt Improvement Method

Nearest Neighbor Algorithm

- Pick any starting node
- Go to the nearest node not yet visited
- Continue from there to the nearest unvisited node
- Repeat this until all points have been visited
- Then return to the starting point.

Nearest Neighbor

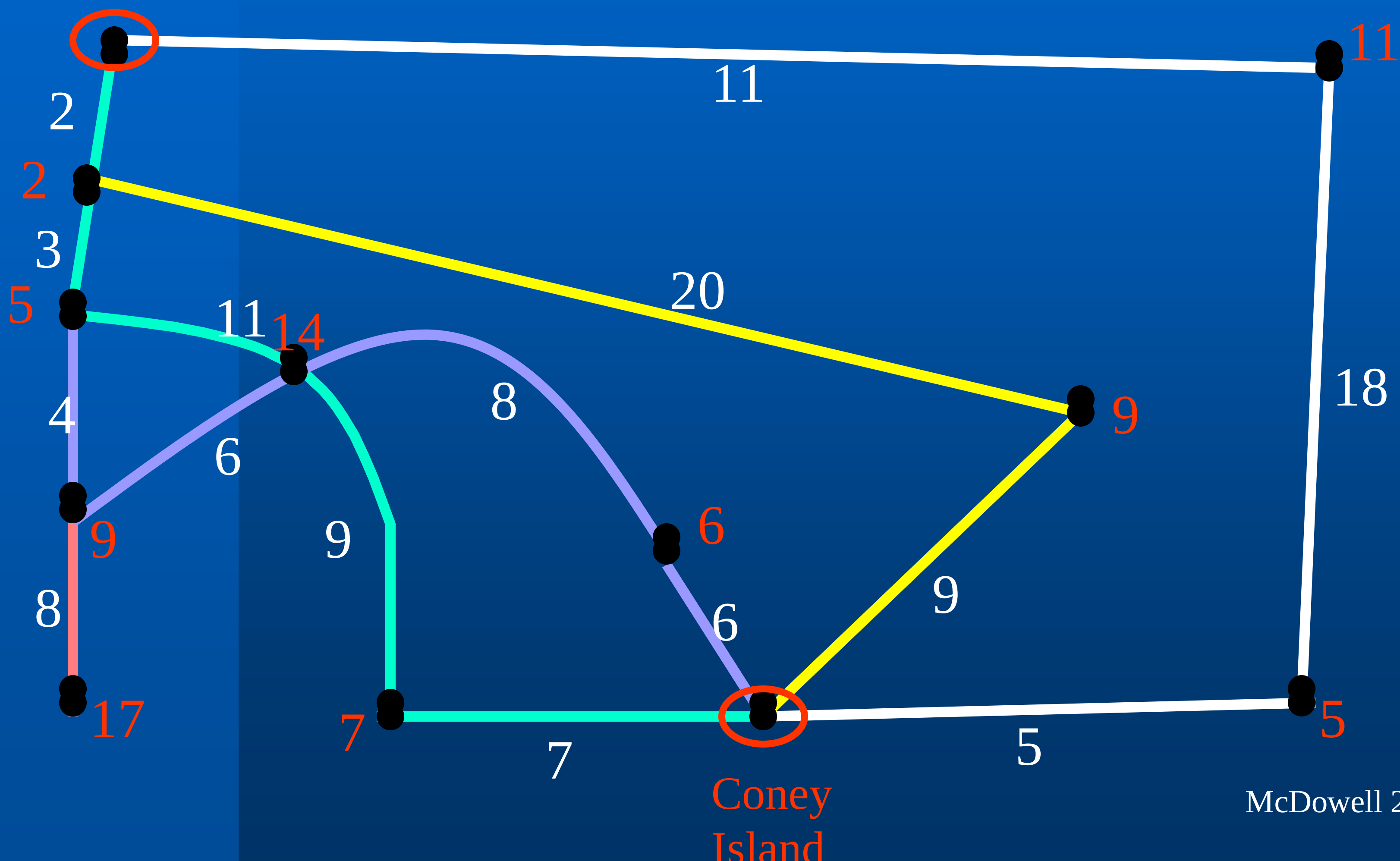


Time
5
11
8
6
4
3
2
11
28
23
16
9
126

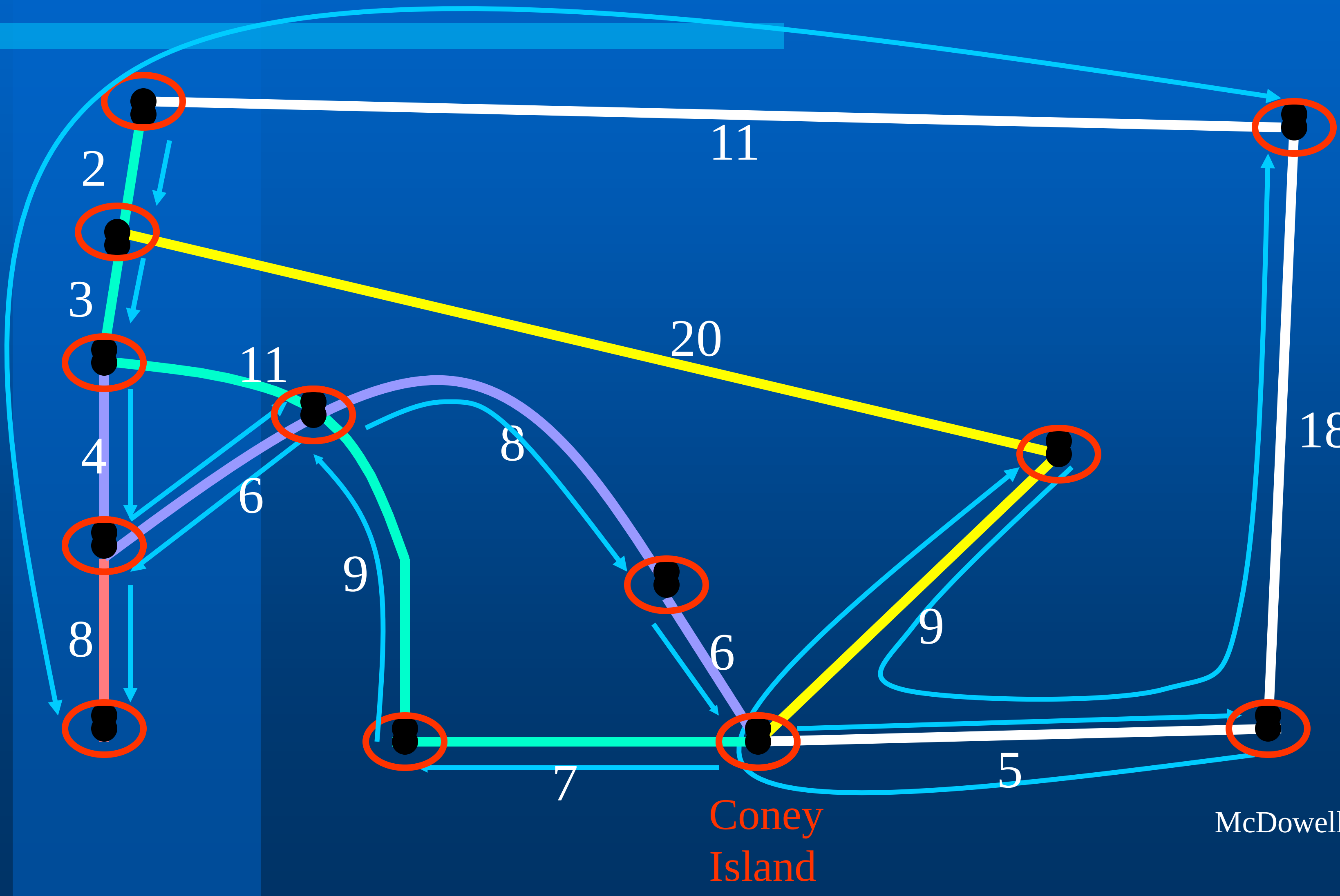
Nearest Insertion Method

- **Start with two nodes that have a high-time tour.**
- **For each uninserted node, figure the minimum time between it and an inserted node.**
- **Insert the node that has the *minimum* time.**

Nearest Insertion



Nearest Insertion

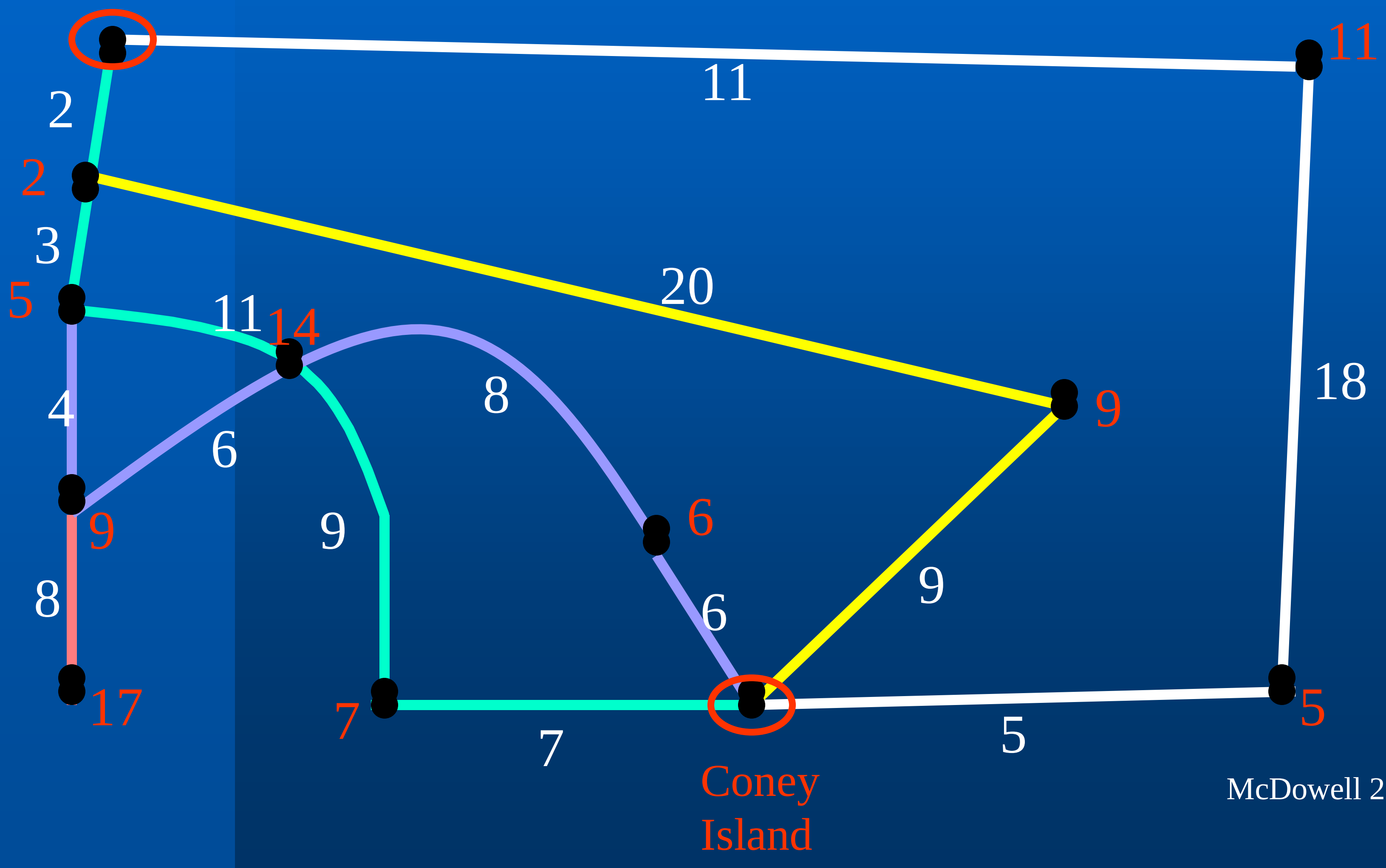


Time
2
3
4
5
6
8
6
7
9
6
8
14
32
110

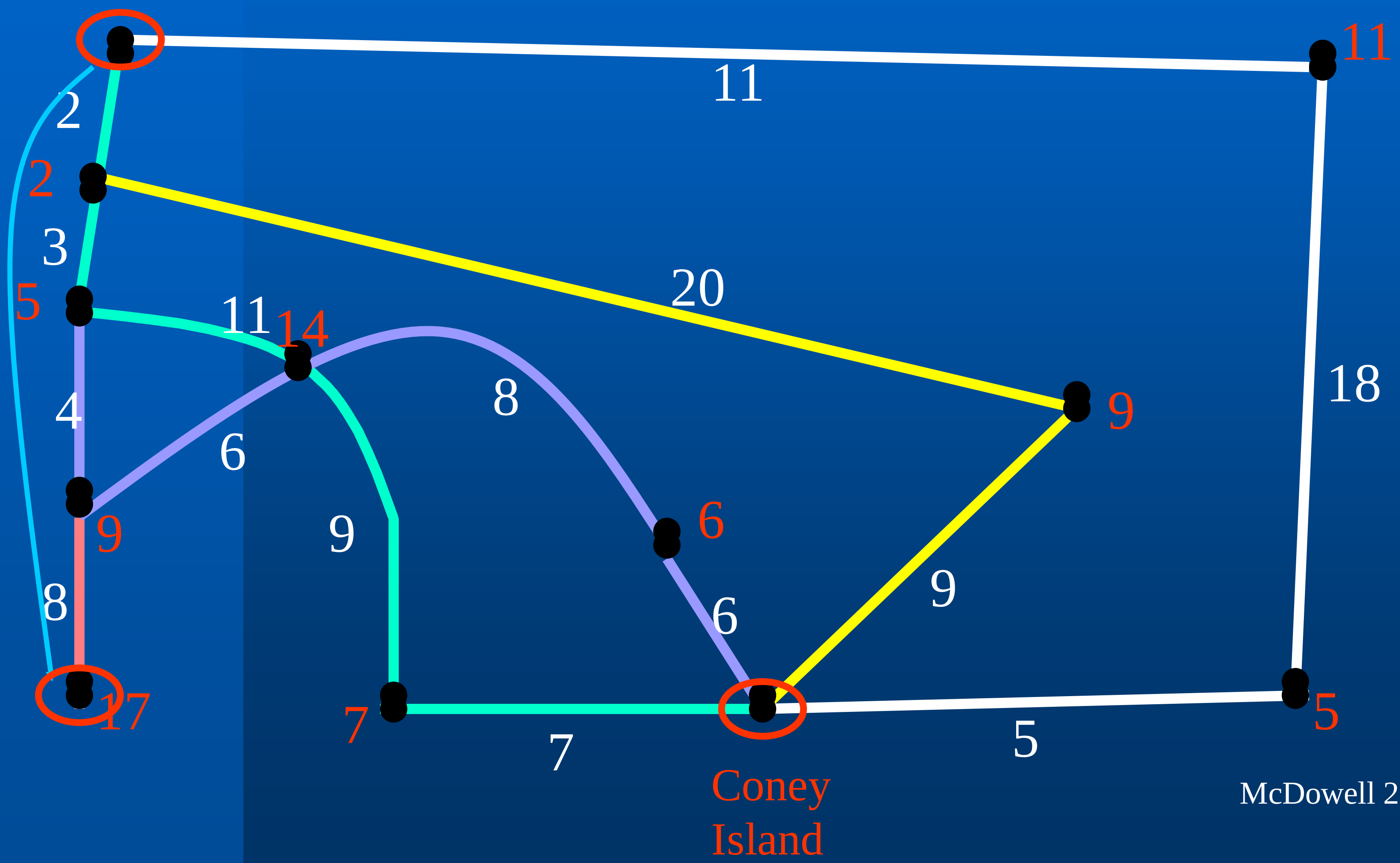
Farthest Insertion Method

- **Start with two nodes that have a high-time tour.**
- **For each uninserted node, figure the minimum time between it and an inserted node.**
- **Insert the node that has the *maximum* time.**

Farthest Insertion



Farthest Insertion

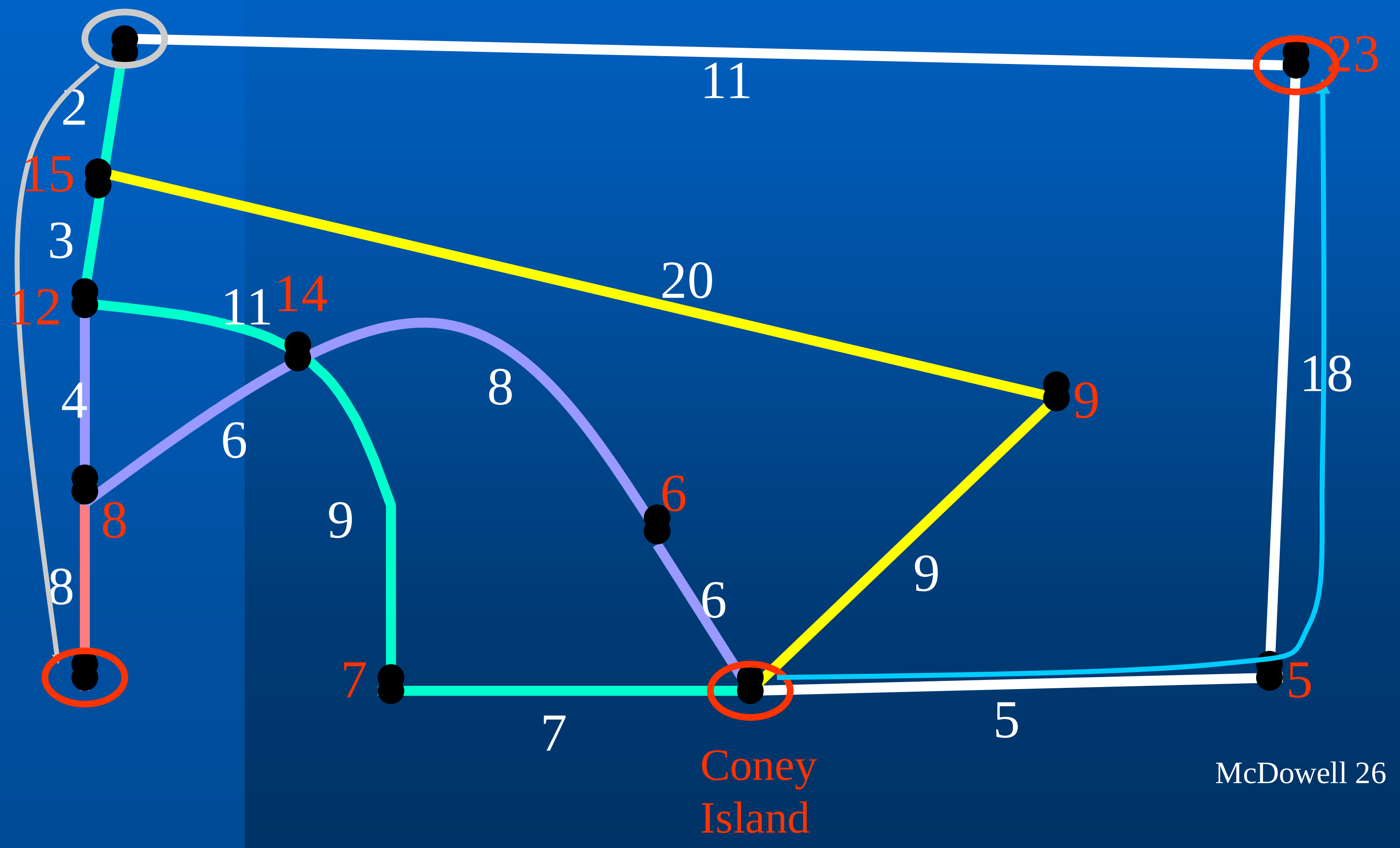


Farthest Insertion

Time

17

23



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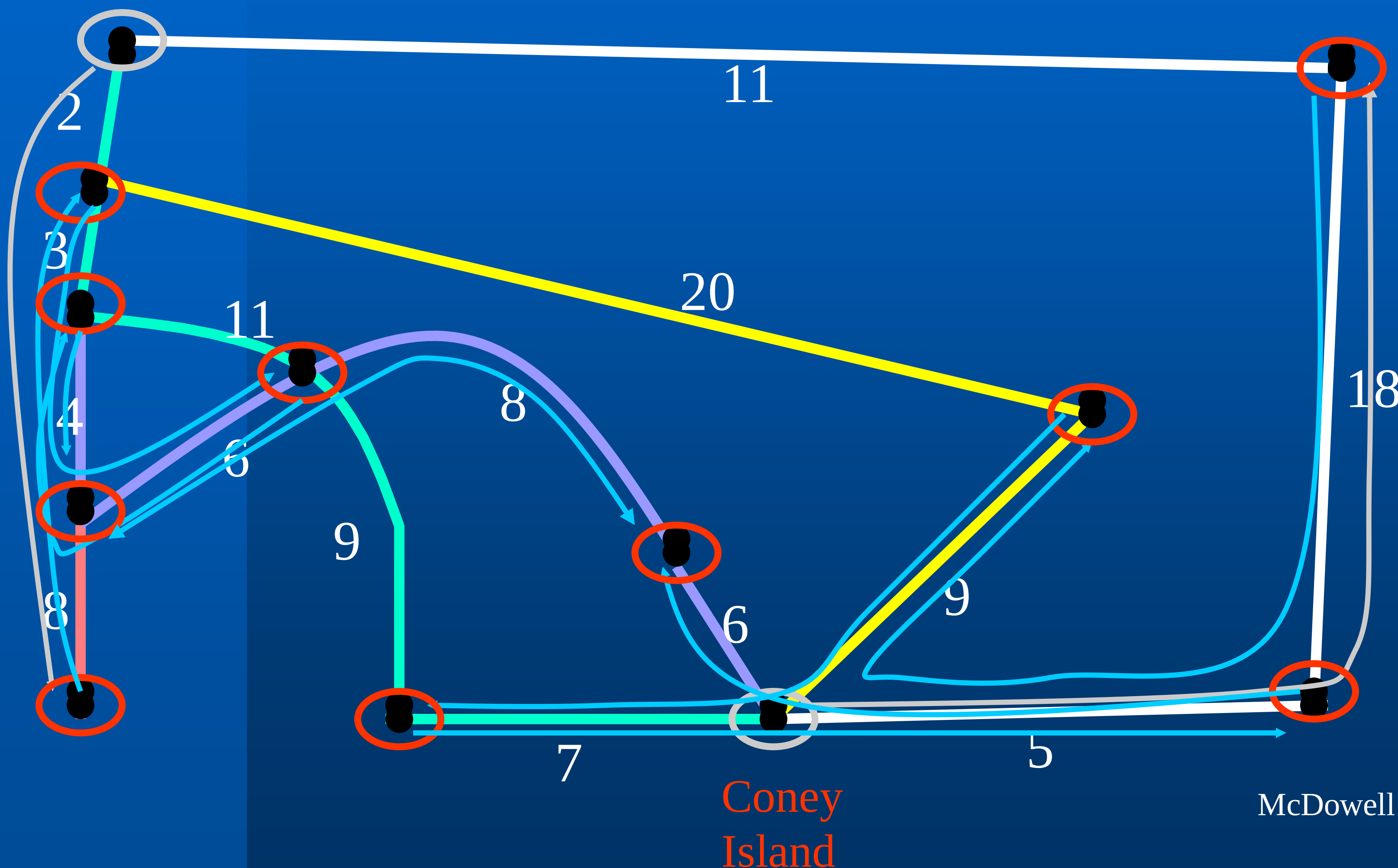
Farthest Insertion



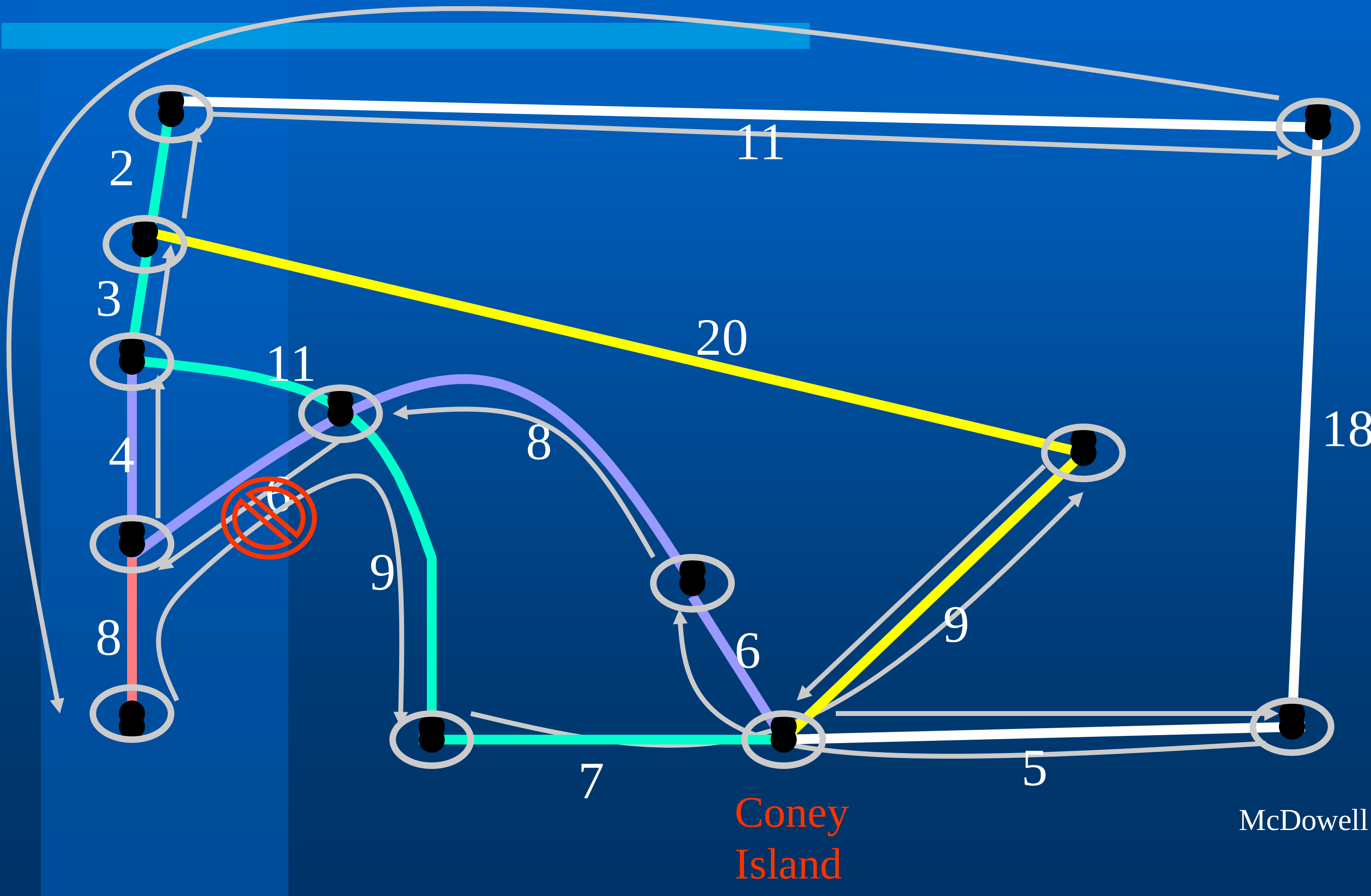
Time
17
23
32
16
15
12
13
10
11
4
14
167

2-Opt Improvement Method

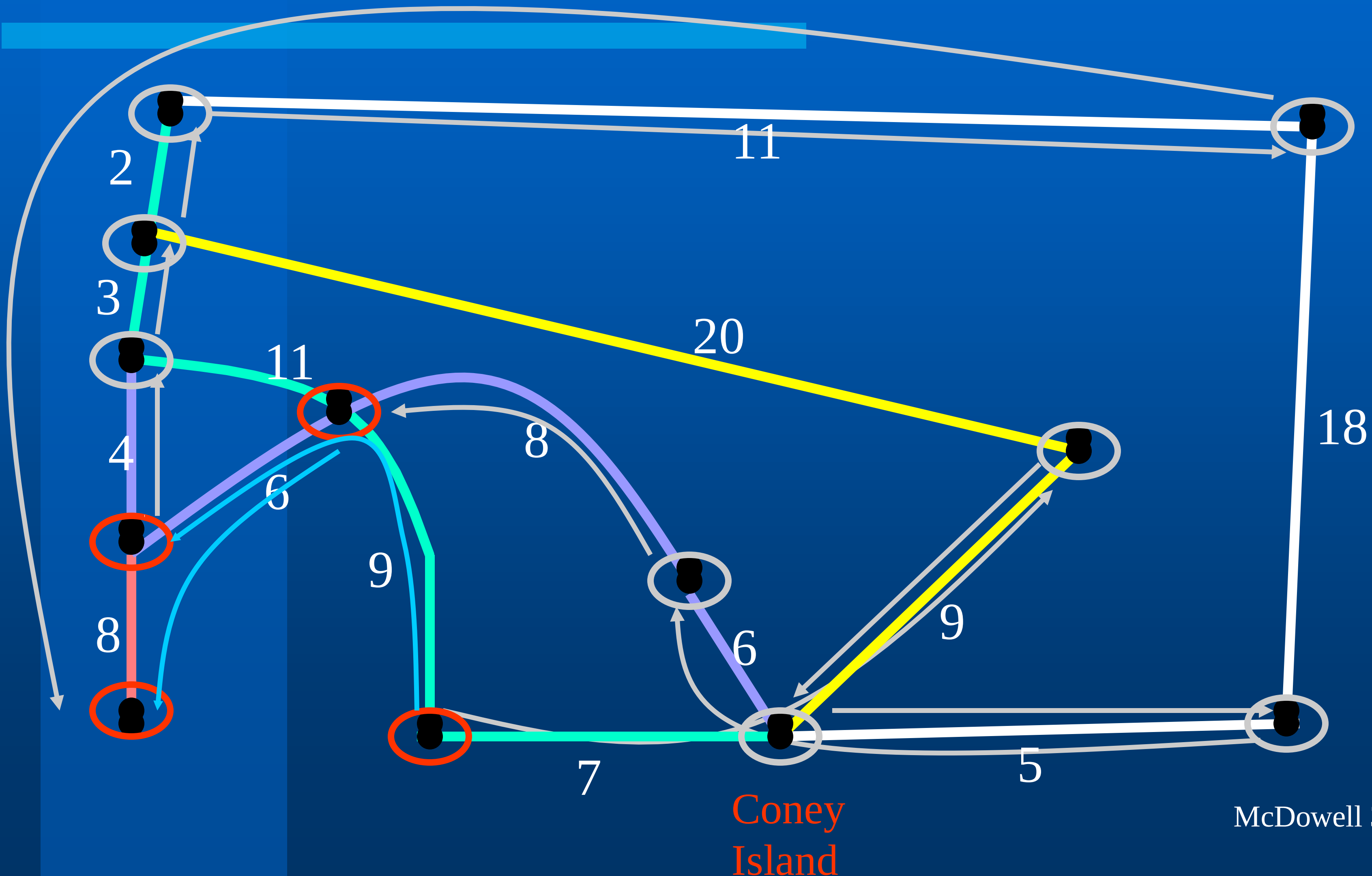
- Consider all edges of a tour.
- Remove two edges between a total of four nodes.
- Connect the start of one deleted edge to the start of the other deleted edge.
- Connect the end of one deleted edge to the end of the other deleted edge.

2-Opt

Time
5
11
8
6
4
3
2
11
28
23
16
9
126

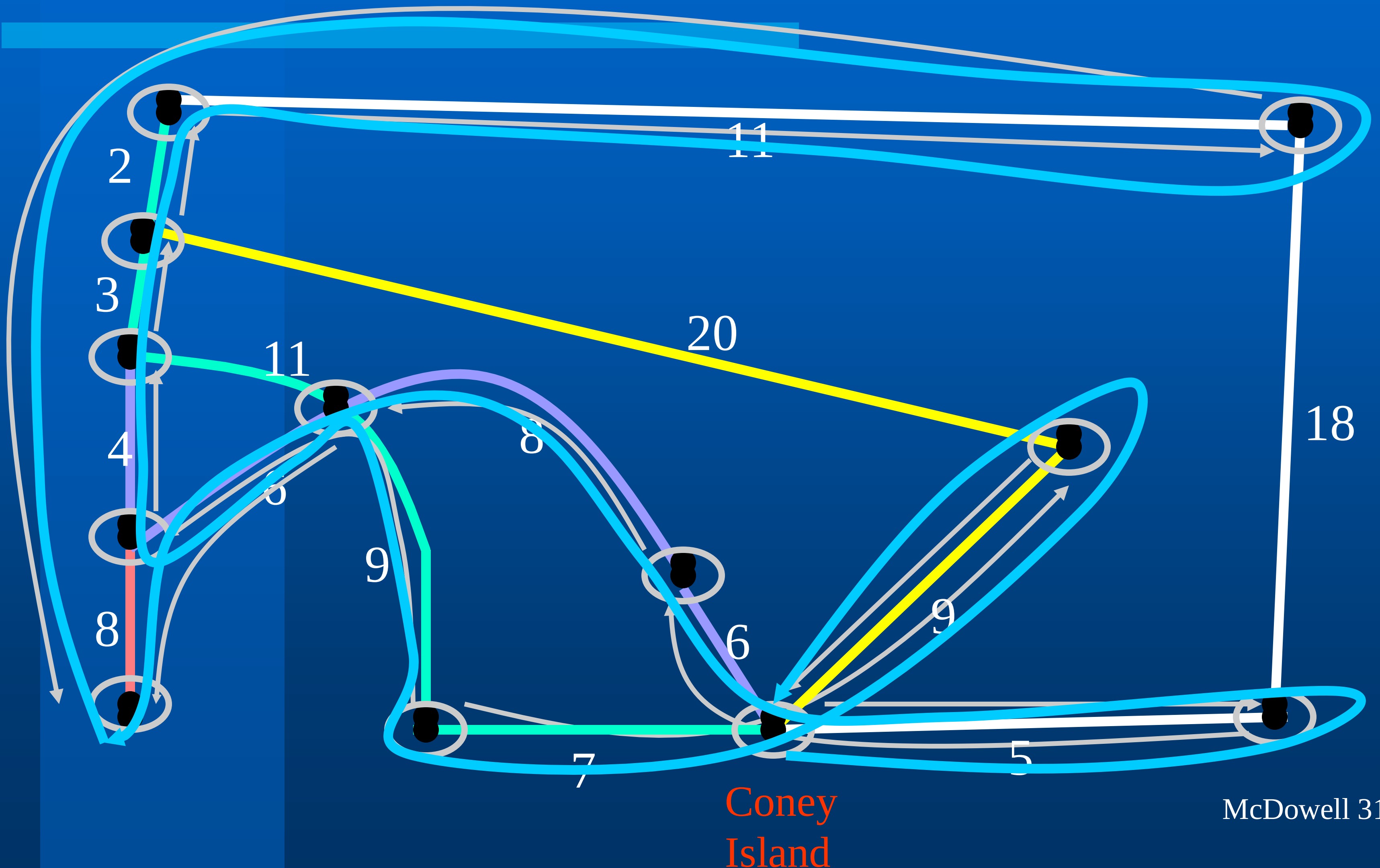


2-Opt

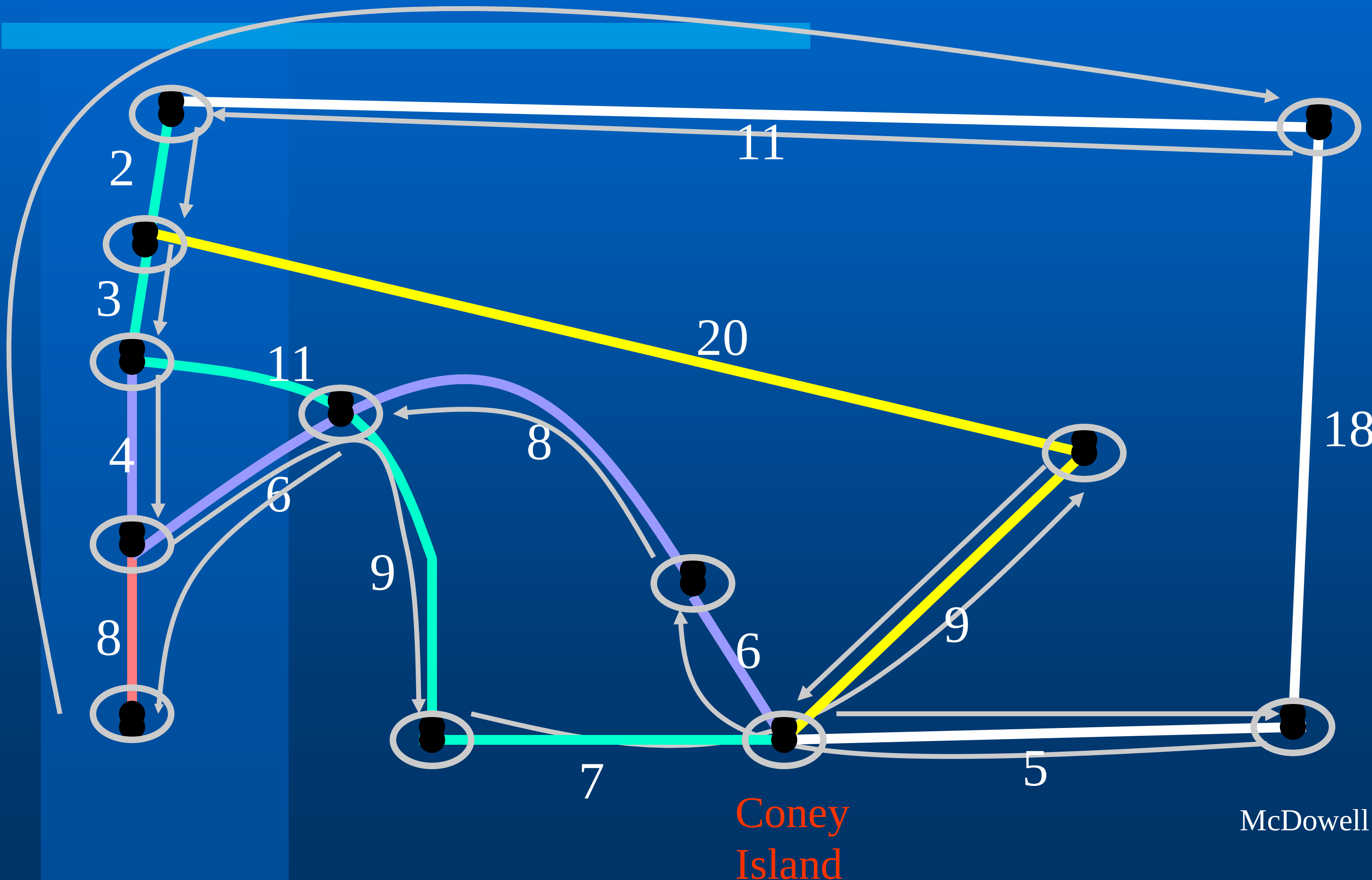


Time

2-Opt



2-Opt



Time
5
11
8
6
8
4
3
2
11
11
2
3
4
8
6
9
7
9
9
126

K-Opt Improvement Method

- Consider all edges of a tour.
- Remove k edges between select nodes.
- Connect the start of one deleted edge to the start of the other deleted edge.
- Connect the end of one deleted edge to the end of the other deleted edge.

Best Solution of Chosen Algorithms

- **Nearest Neighbor = 126 minutes**
- **Nearest Insertion = 110 minutes**
- **Farthest Insertion = 167 minutes**
- **2-Opt Improvement = 126 minutes**

- **Note: This will not always be the case.**

Conclusion

- There is no proven efficient algorithm for the Traveling Salesman Problem. There are algorithms available, but not one always produces the minimum time. The algorithms can give insight to problems and how we relate to them in the real world.