FRACTALS

Patricia Rush

Senior Seminar Project

Chaos

- Chaos is apparently unpredictable behavior arising in a deterministic system because of great sensitivity to initial conditions.
- The idea that seemingly chaotic and disconnected systems actually do have order.

Quote

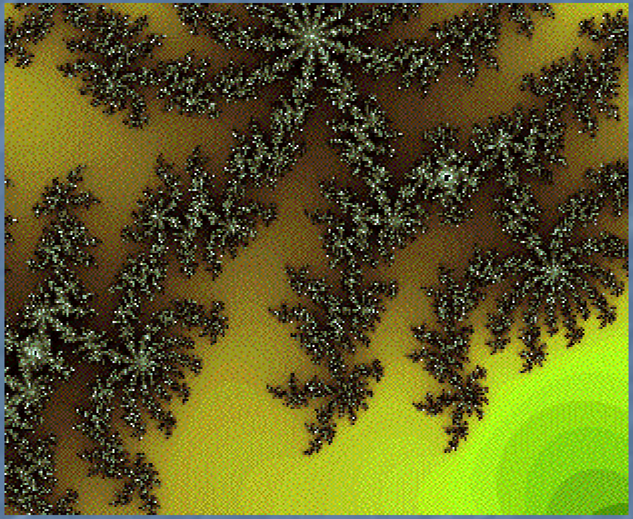
"Can the flap of a butterfly's wing stir up a tornado in Texas?"

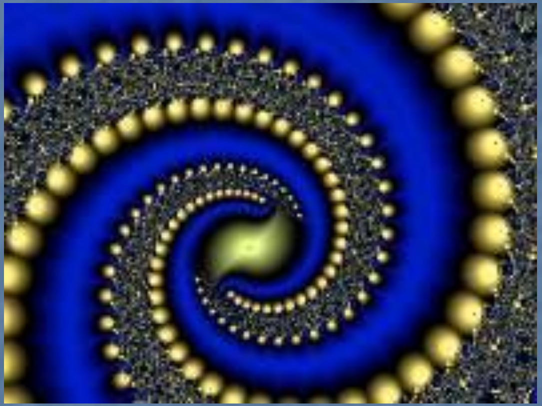
-Edward Lorenz

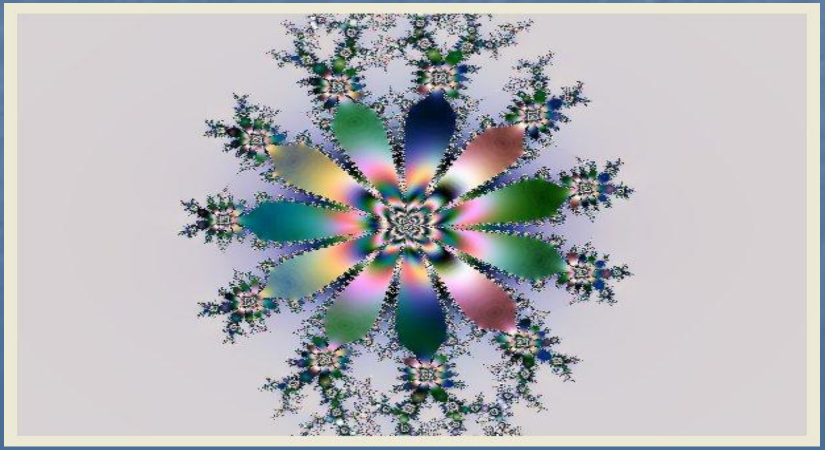
Role of the mathematician

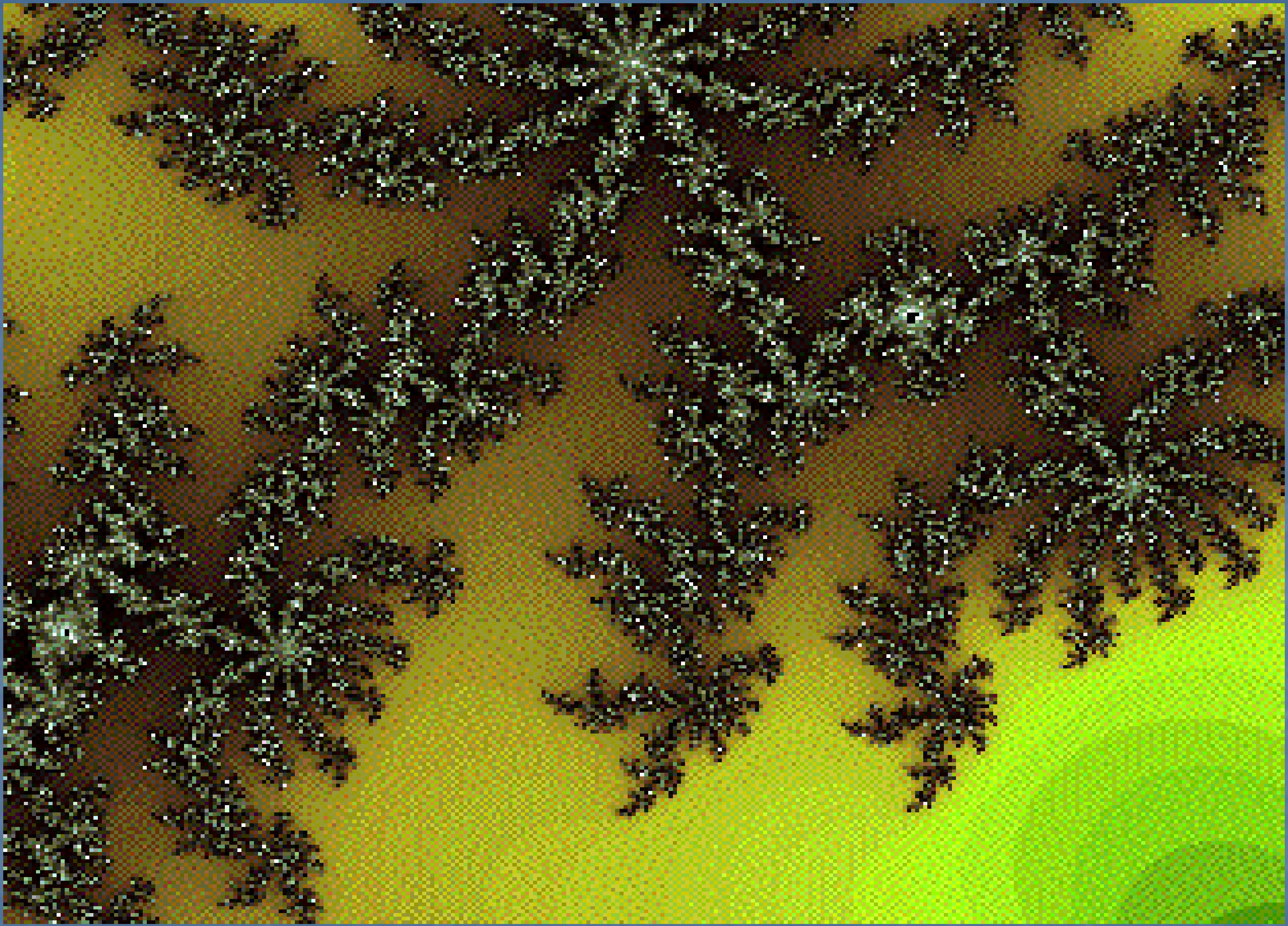
- This idea intrigued scientists and mathematicians.
- If they could find some pattern, then maybe they could predict things such as weather.

What is in common?

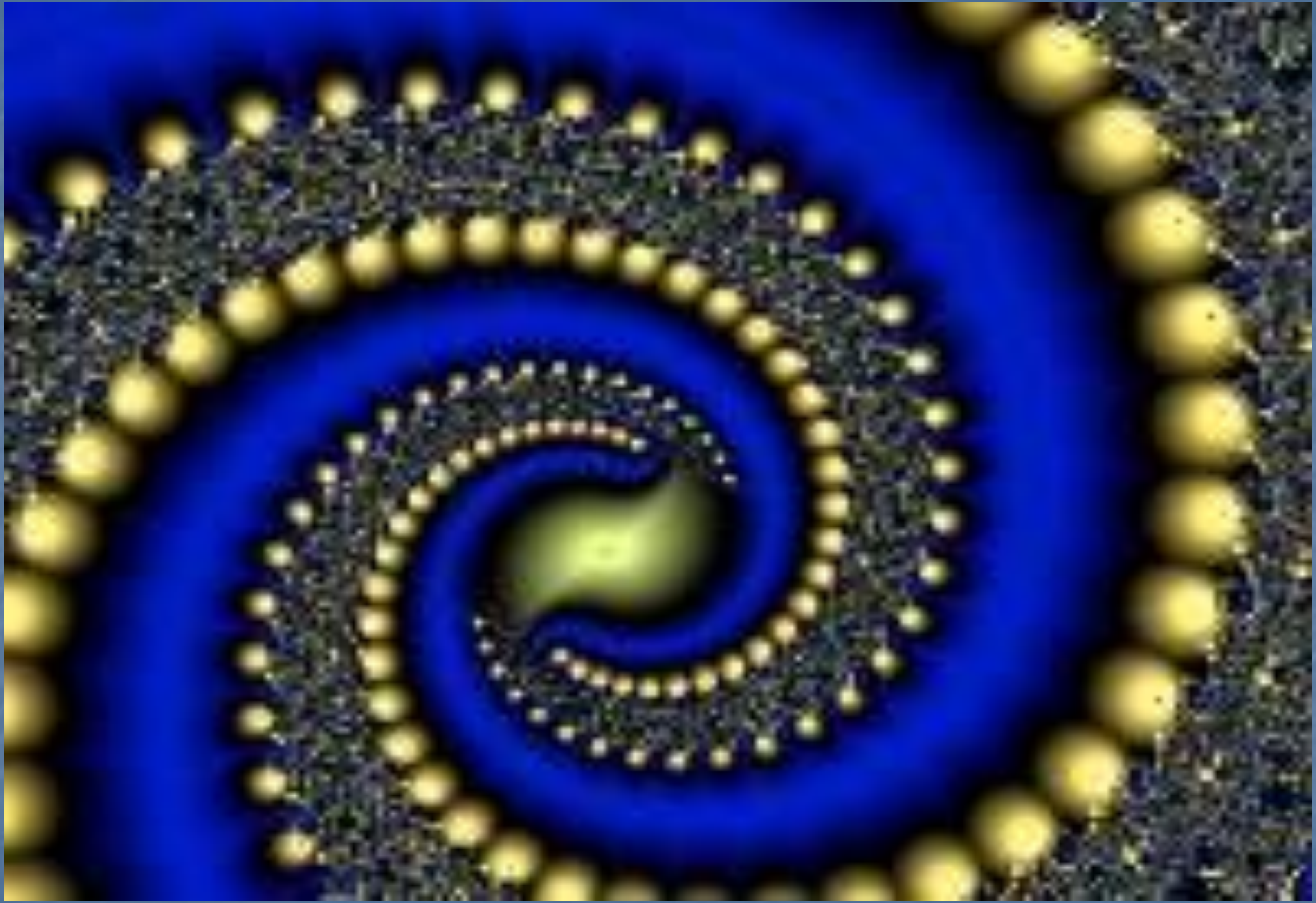




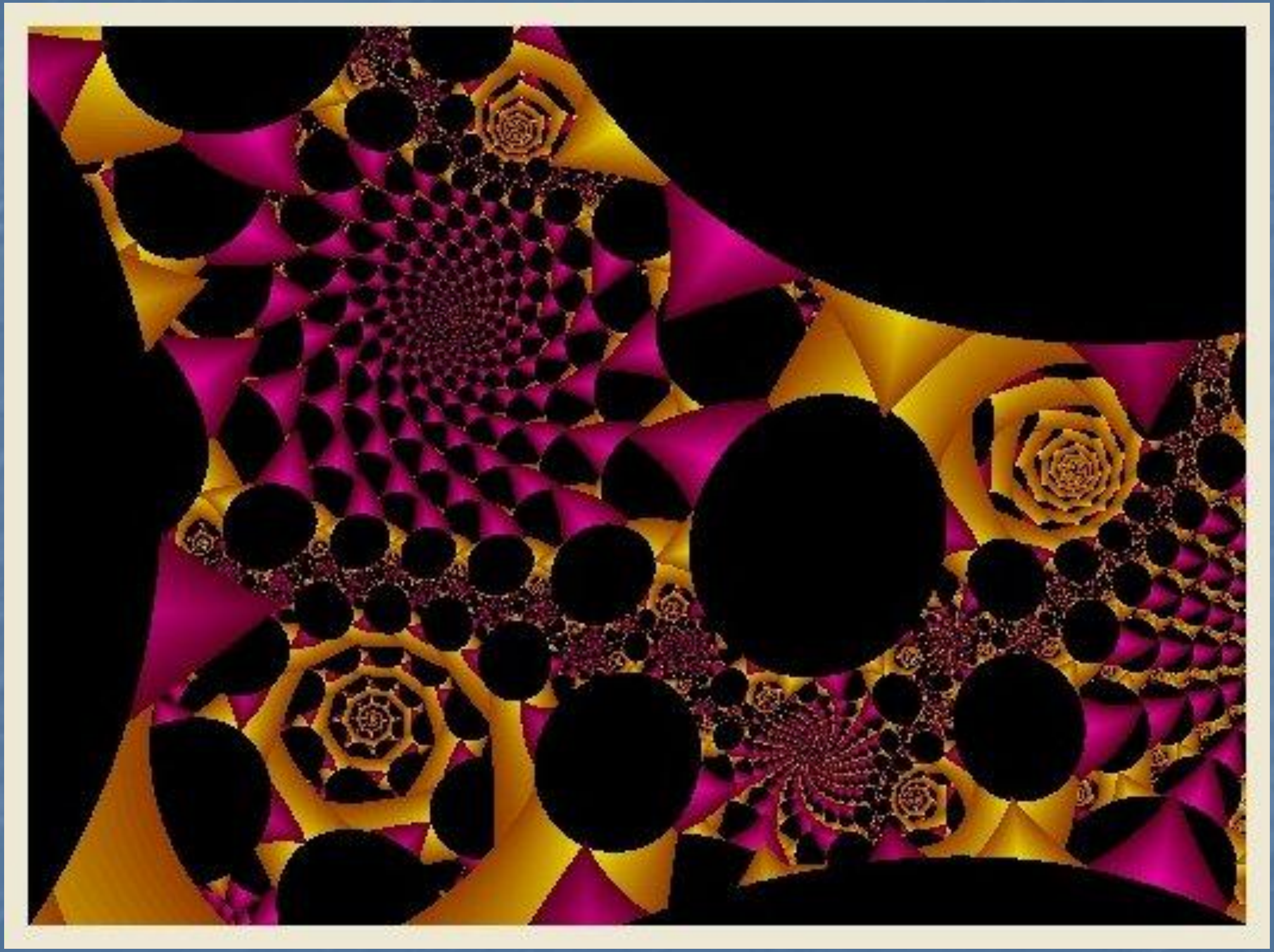




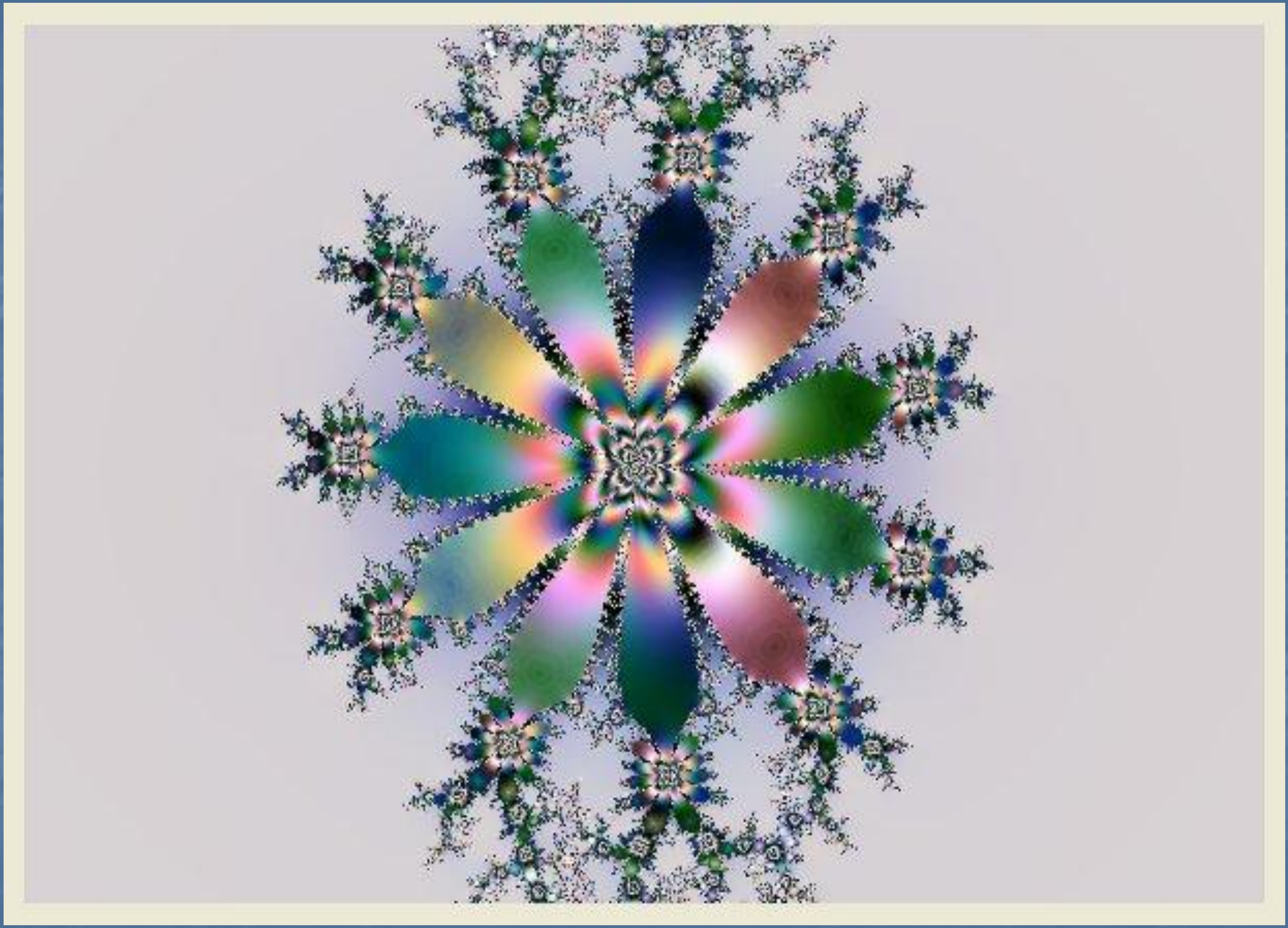
http://archive.ncsa.uiuc.edu/Edu/Fractal/Fractal_Home.html



http://home.wtal.de/spiriteye/fractal/



http://fourart.homestead.com/gal3.html



http://fourart.homestead.com/gal3.html

What in the world is a Fractal?

Named by Mandelbrot in 1975, from Latin fractus meaning "to break".

- A fractal is a geometrical figure in which an identical motif repeats itself on an ever diminishing scale.
- A fractal is an image that repeats itself over and over again within itself.

What do fractals have to do with chaos?

Fractals have some of the same chaotic characteristics.

Fractals are:

- Sensitive to small changes
- Unpredictable
- Appear chaotic, even though they were created using non-chaotic equations.

Key Ideas

- Recursion
- Iterations
- Self-Similarity
- Fractional Dimension

Why in Mathematics?

- Mathematical equations can be assigned to explain the recurring nature of the fractals.

Fractals made simple

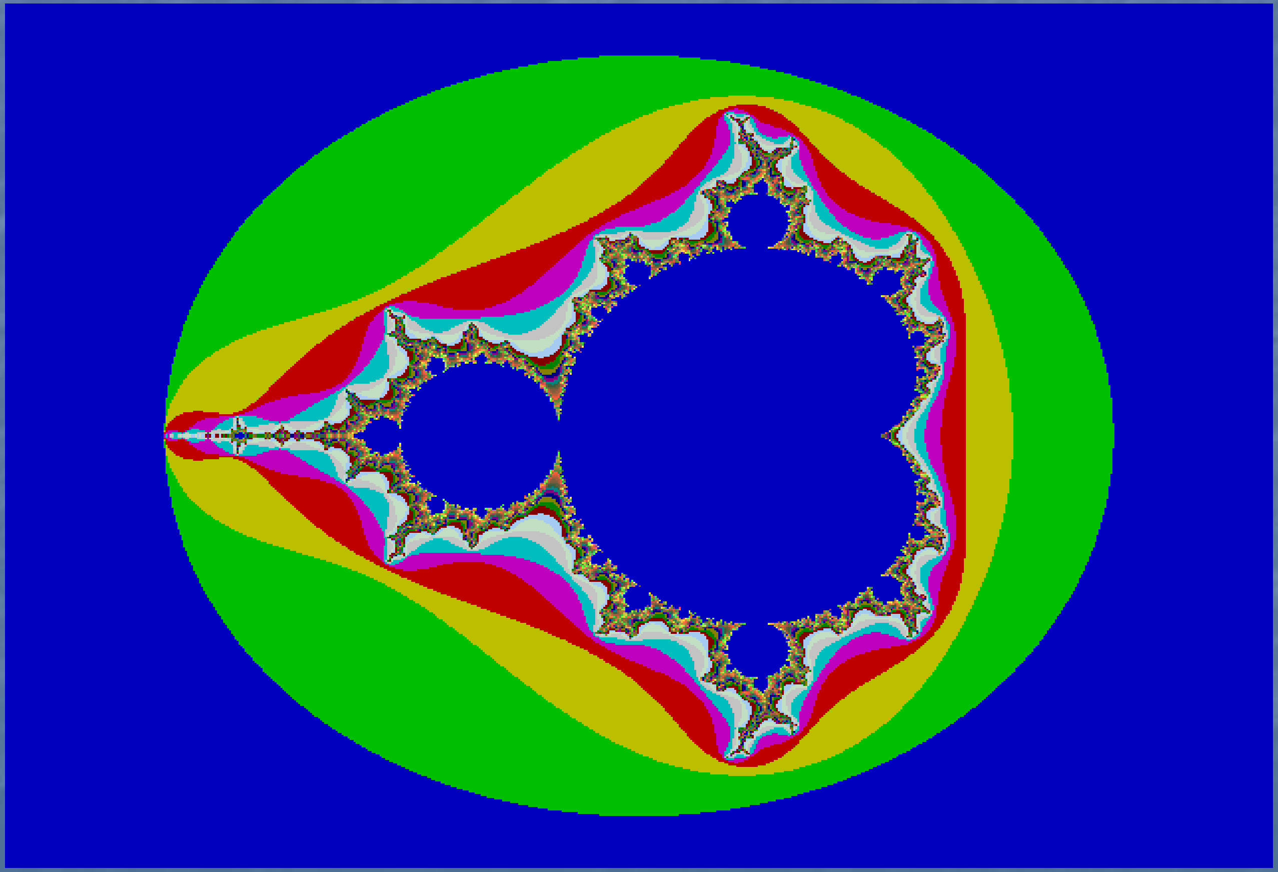
Key concepts:

- ✓ Functions
- ✓ Graphing
- ✓ Complex numbers

Background Information

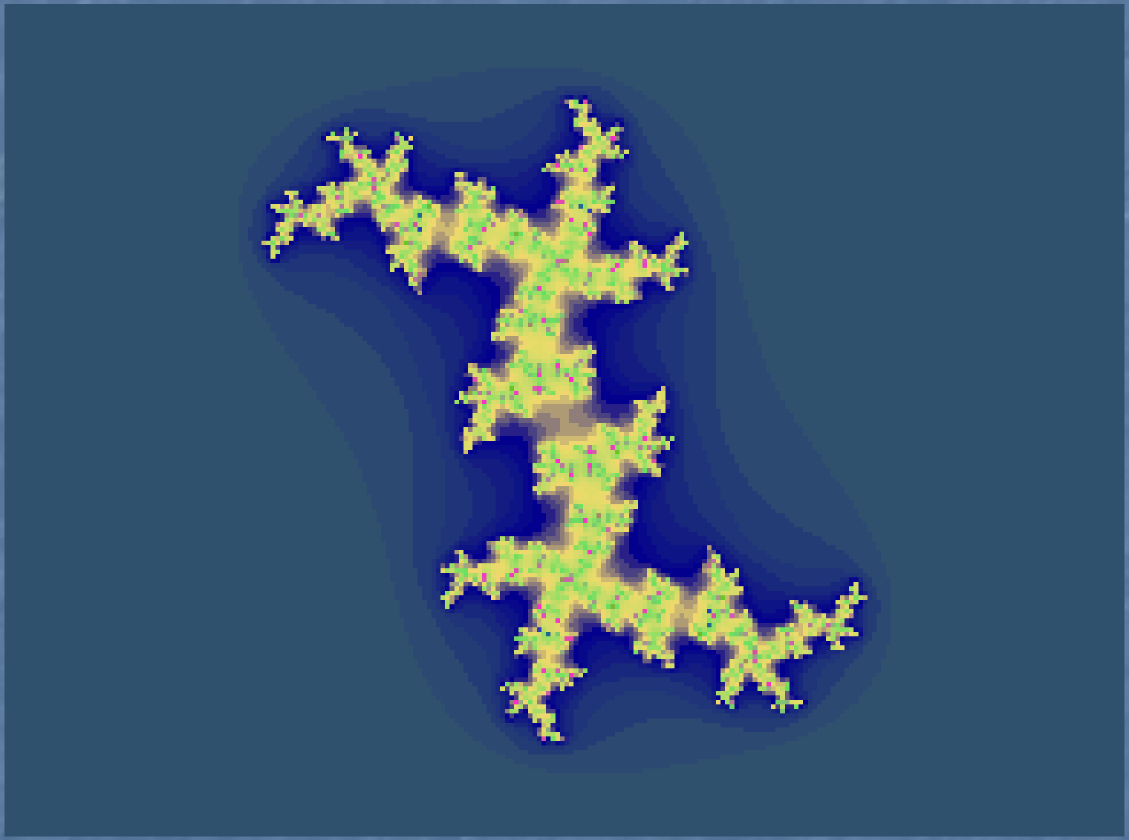
- Fractals use complex numbers instead of the familiar (x,y) coordinates.
- Use x+iy instead
  - X = real
  - Y = imaginary
- Use a different coordinate plane
  - X-axis = real numbers
  - Y- axis = imaginary numbers

The Mandelbrot



Internet site: http://library.thinkquest.org/3703/pages/chaos/html.

The Julia Set



http://spanky.triumf.ca/www/fractint/julia_type.html

Using Computers to Generate

Using the equation : $z_n{}^2 = z_{n-1}{}^2 + c$

- Pick a point: 2+1i
- Pick a constant: c=0
- Substitute in the equation:

  $z_n = z_{2+i} = (2+i)\ (2+i) + 0 =$

  $4+2i+2i+i^2$

$=3+4i$

Recursion

- Now that we have a new value we will execute the function with this value. This idea is called recursion. This can also be called iterating the problem. The more iterations, the more complex.
- Remember to keep the constant the same for each different Julia Set.

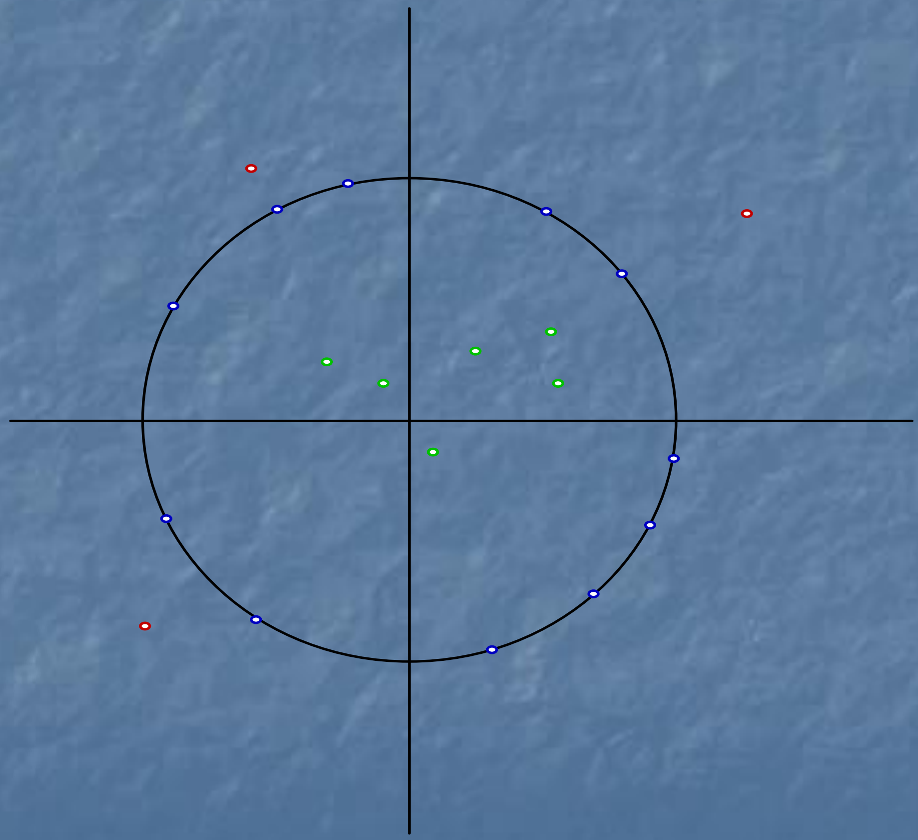
Recursion in Action

- With the same formula as before:

  $z_n^2 = z_{n-1}^2 + c$

- Substitute our new value of 3+4i
  - $z_n^2 = (3+4i)^2 + 0$
  - $= (3+4i)(3+4i)+0$
  - $= 9+12i+12i+16i^2$
  - $= -7+24i$

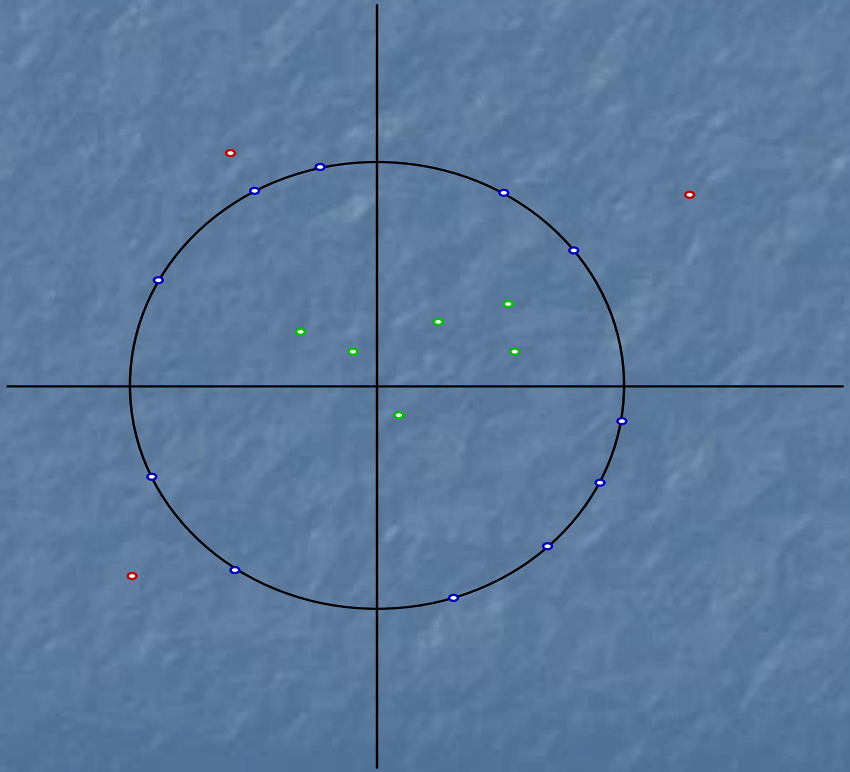
Julia Set for c=0



Courtesy of *Geometer's Sketchpad*

Key terms in the complex plane

- Escape Set – points for which the iteration produces values that are unbounded.
- Prisoner Set – points for which the iteration produces values that are bounded.
- Boundary – points for which every neighborhood contains points from both the escape and prisoner sets.



- Escape set
- Prisoner Set
- Boundary Set

What's next?

- All the points created with a constant form a Julia Set.
- The Julia Sets that are connected are in the Mandelbrot set, those disconnected are not.

Definitions

- A set is called connected provided it cannot be decomposed into two disjoint, non-empty sets.
- A set is called disconnected if it can be decomposed into disjoint parts.

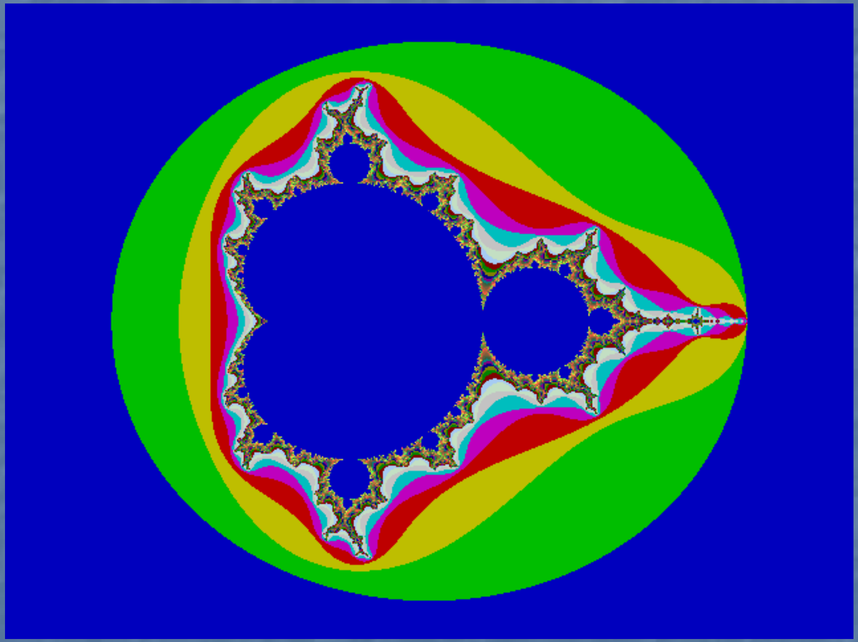
Examples

Connected

Disconnected

http://www.cs.unca.edu/~mcclure/cgi-bin/julia.cgi?c=-1.298+-0.6413333333333331i

Now what?



- The areas that are dark blue are the corresponding connected Julia sets.
- All other areas are the corresponding disconnected Julia Sets.

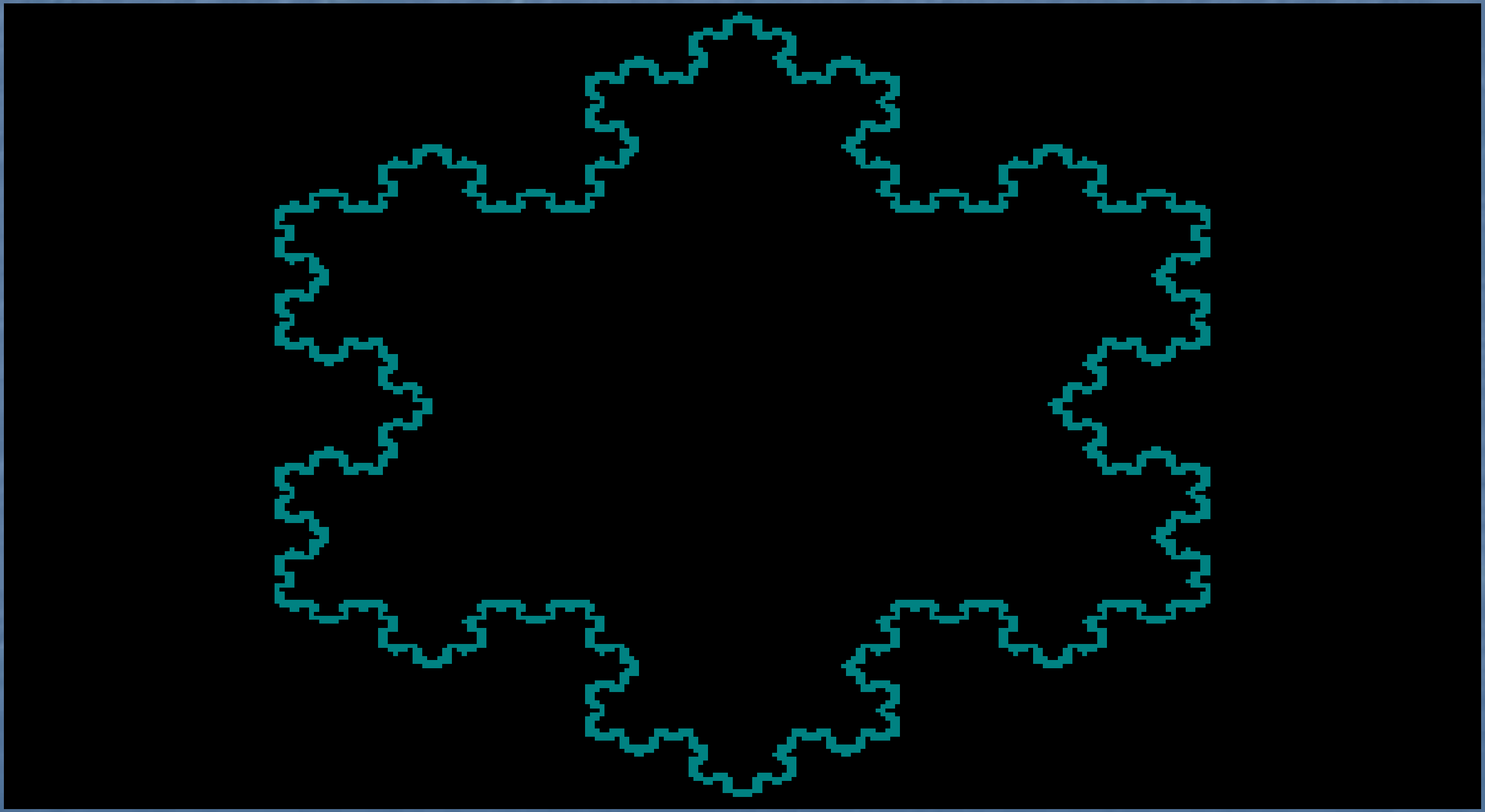
Are the Colors significant?

YES!

The colors tell whether the point is in the set or not.

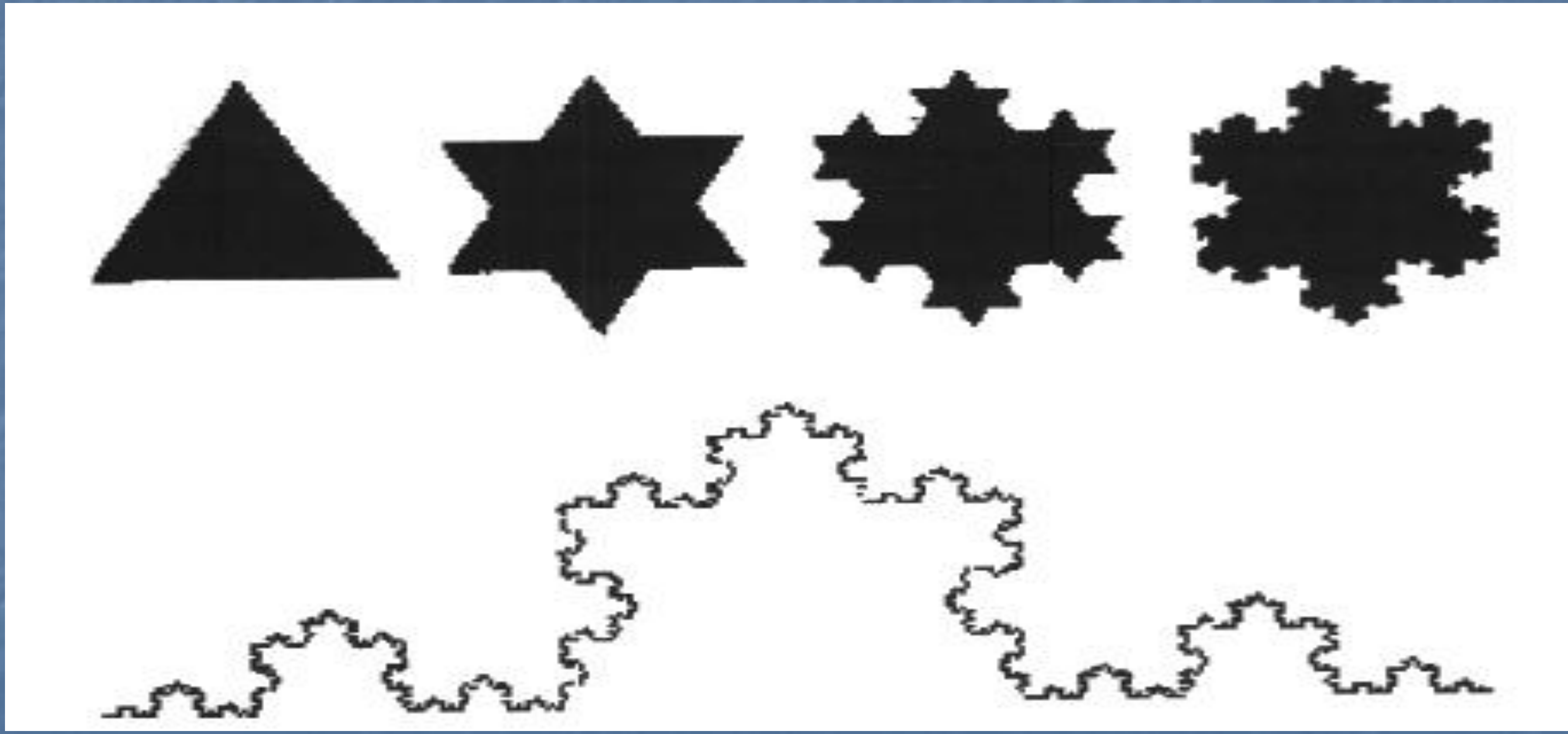
The different colors are symbolic of the different number of iterations.

The Koch Curve



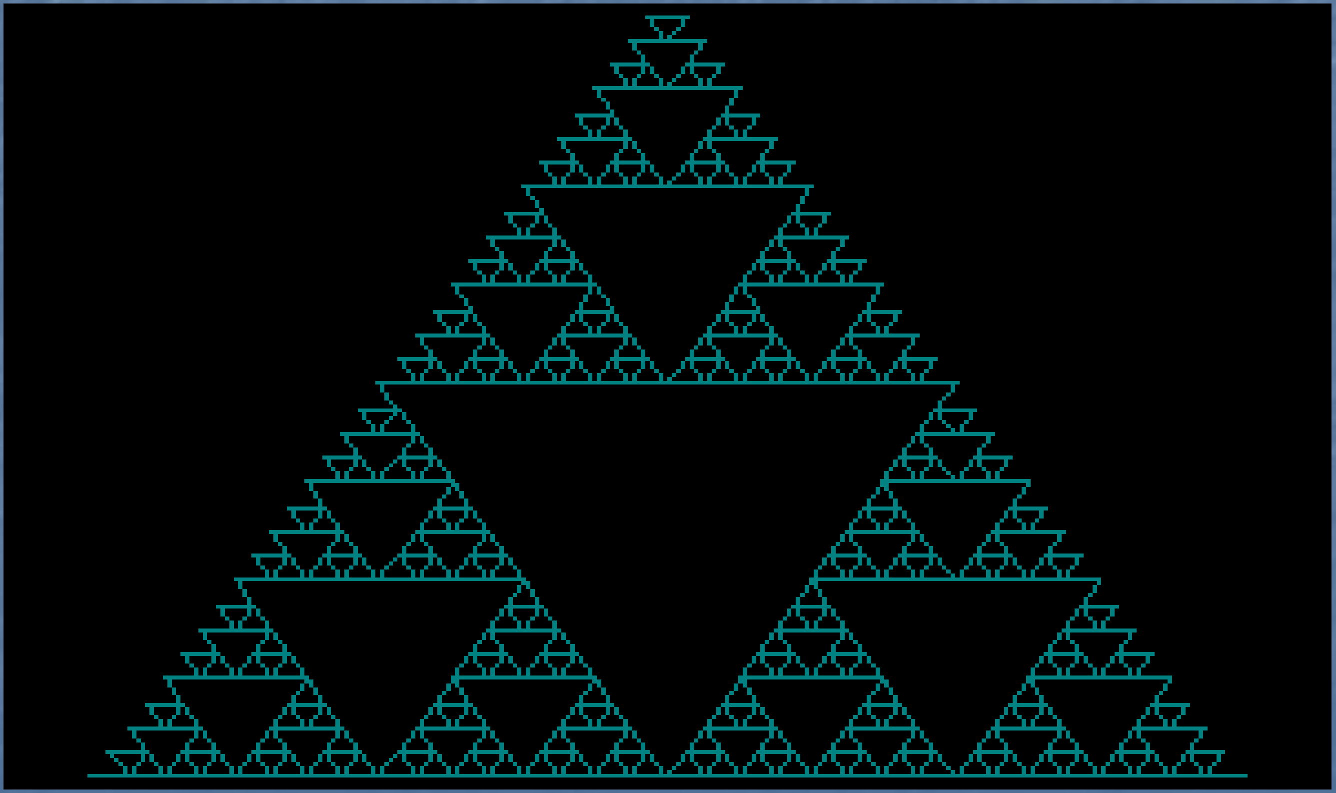
Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html

The Koch Curve



http://emmy.dartmouth.edu/~c18w99/webpages/nkk/koch.html

Sierpinski's Triangle



Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html

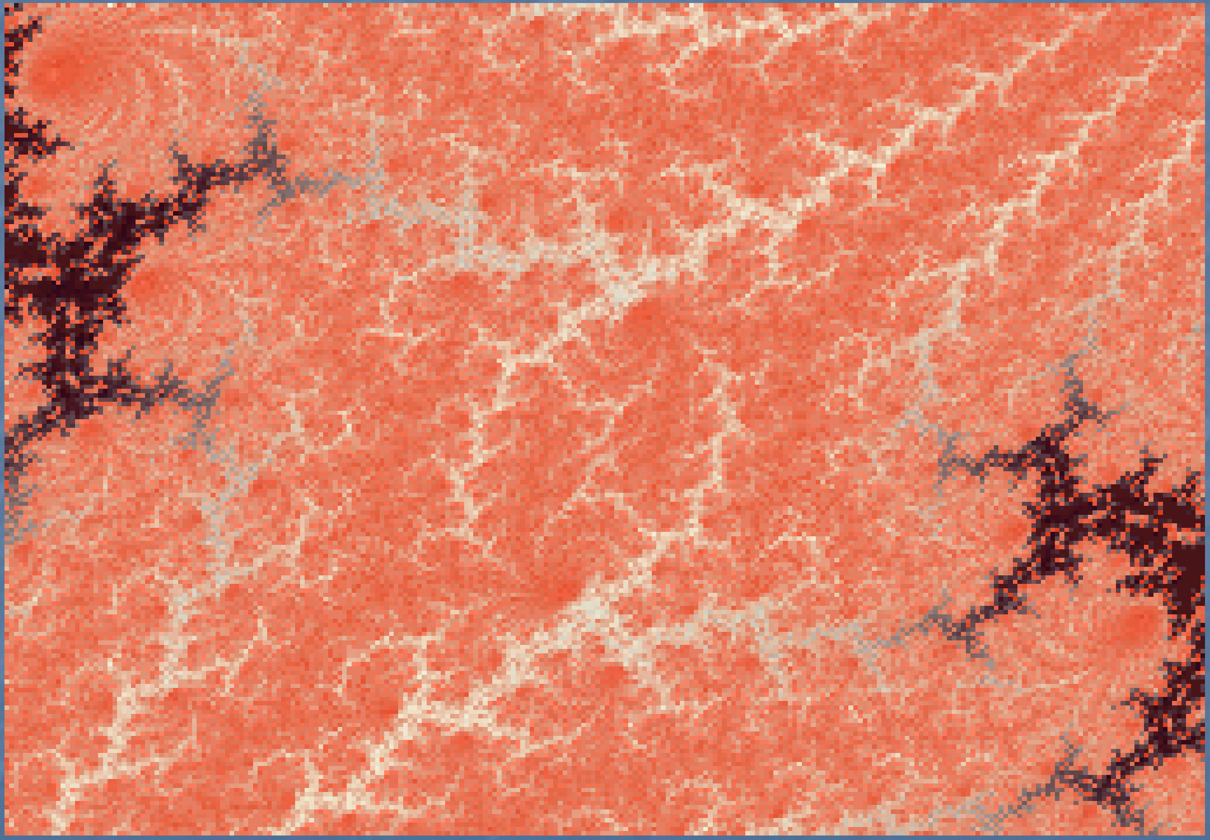
Fractals in the "Real World"

- Human Body
- Nature
- Food
- Landscapes
- Coastlines
- Stock Market
- Weather
- Etc.

The Human Body

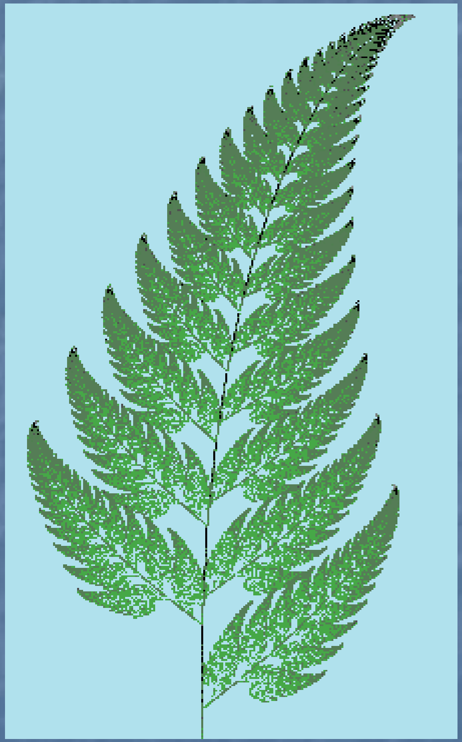
- Neurons
- Cells
- Pathogens
- Brain
- Blood vessels

Capillaries



http://spanky.triumf.ca/www/other_images.html

Ferns and other Plants

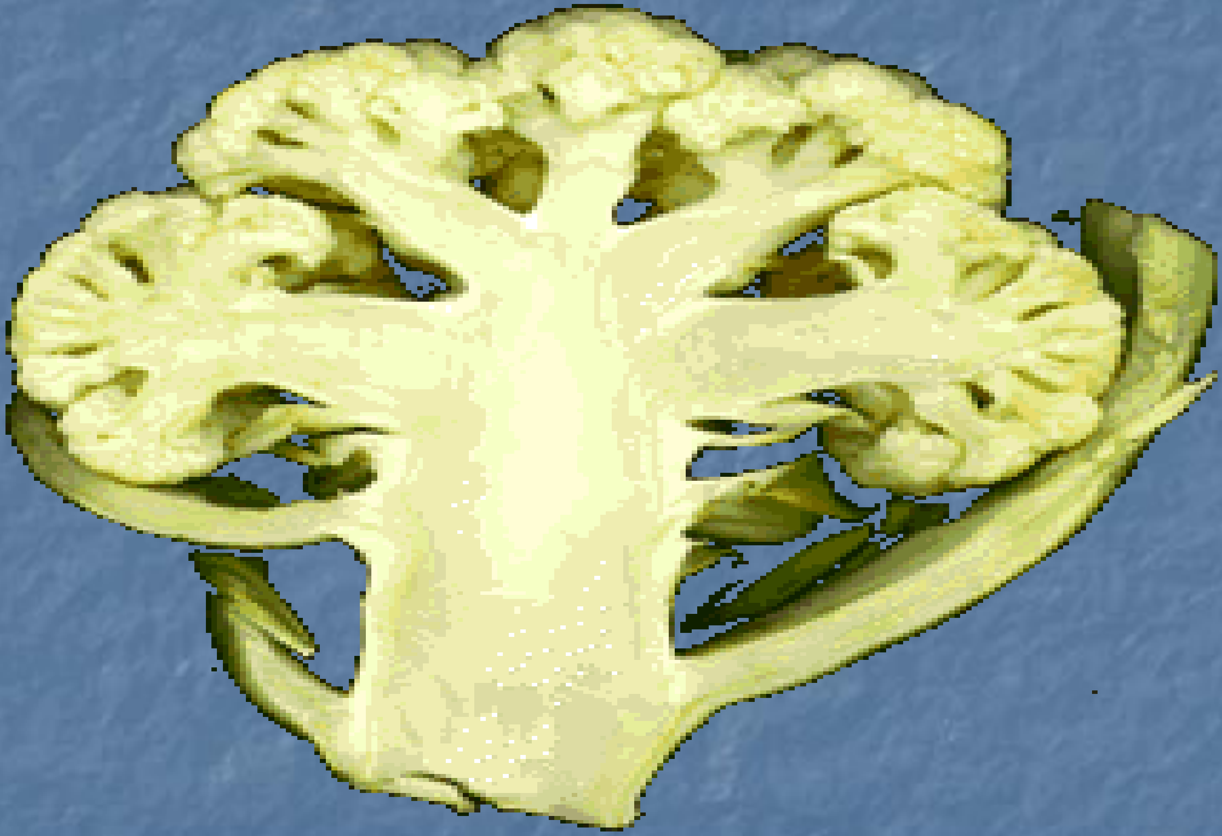


http://www.mste.uiuc.edu/courses/ci330ms/eilken/unitnatu.HTML

Trees

- Branches
- Leaves

Cauliflower

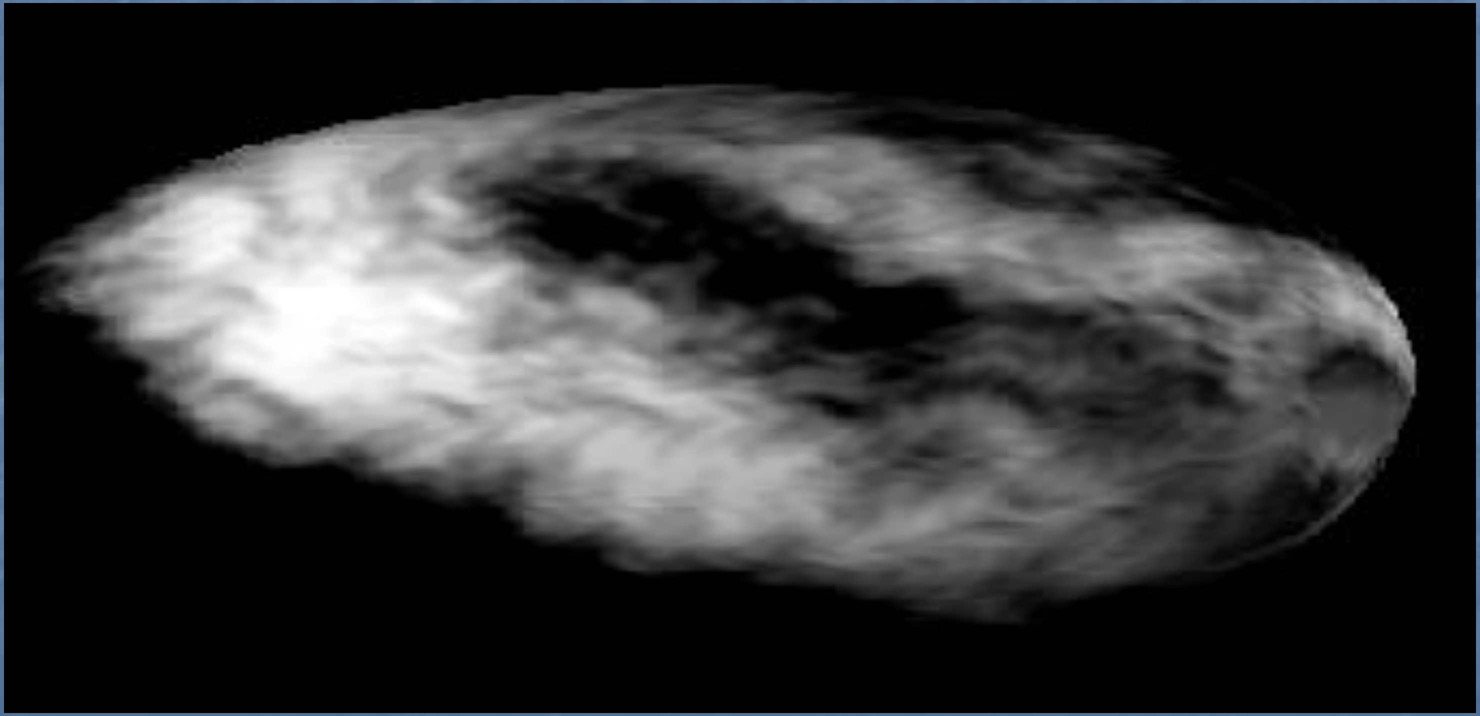


http://cosmopolis.com/df/fractals-exhibit.html

Lightning

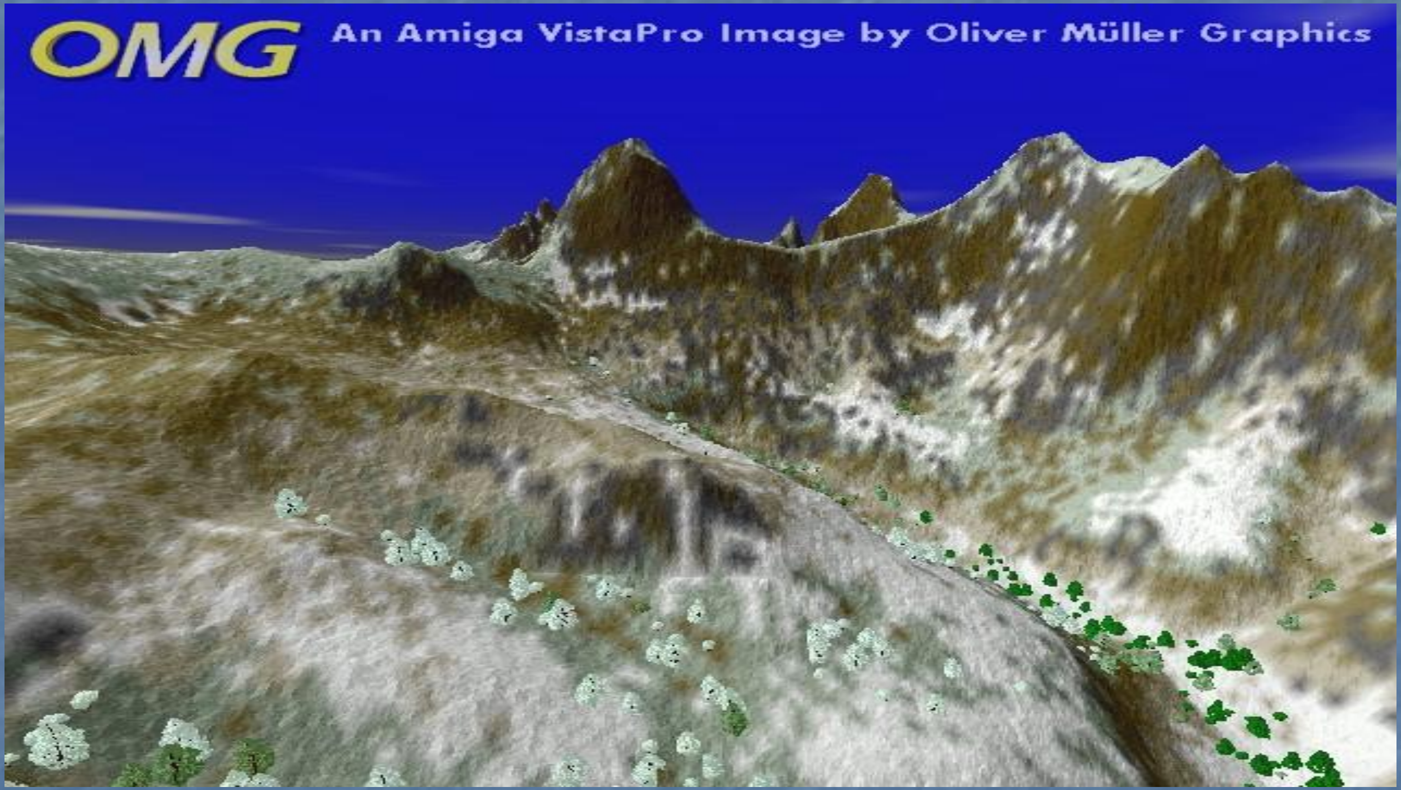
http://www.redflagsdaily.com/harmony.html

Fractal-generated clouds



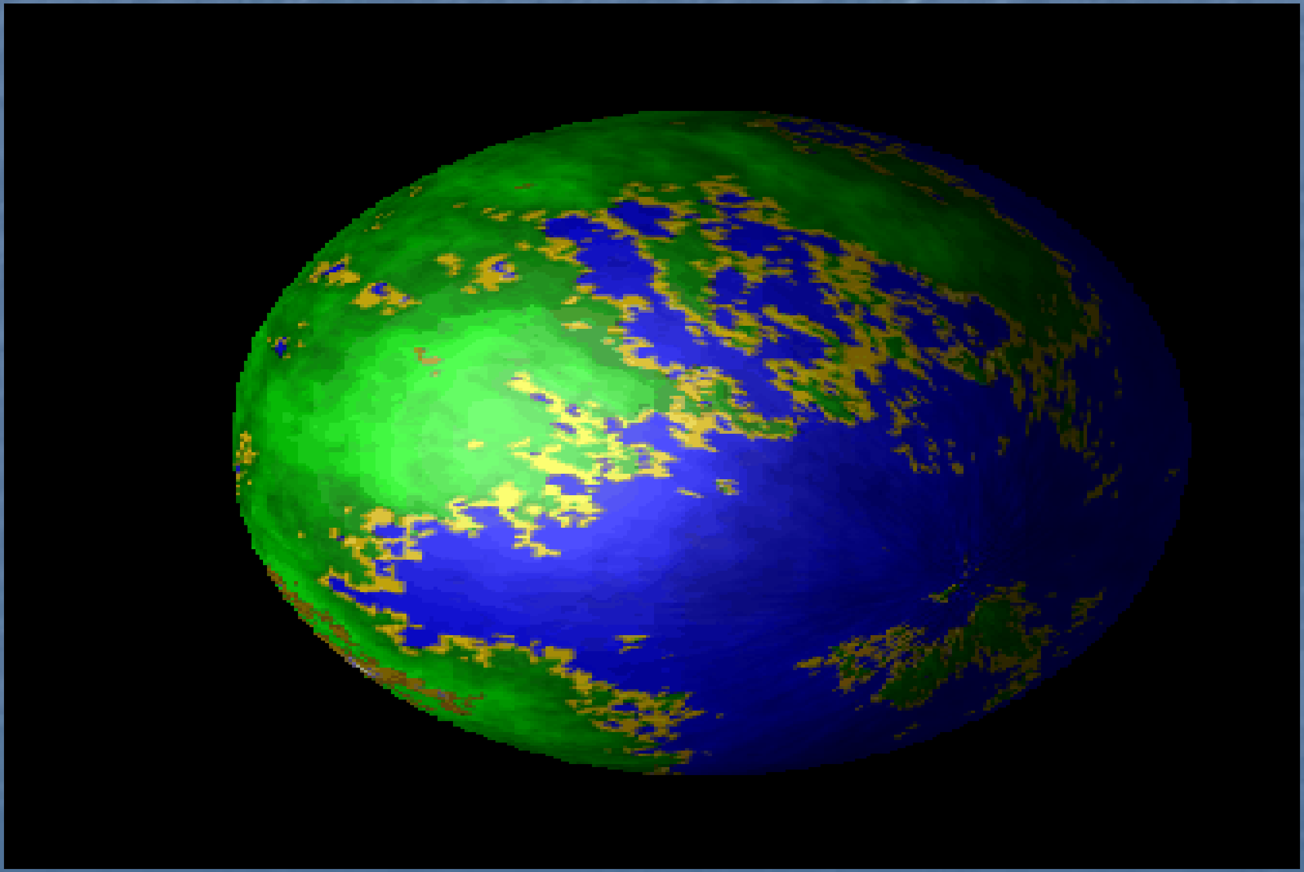
http://www.kcsd.k12.pa.us/~projects/fractal/pics.html

Mountain Range



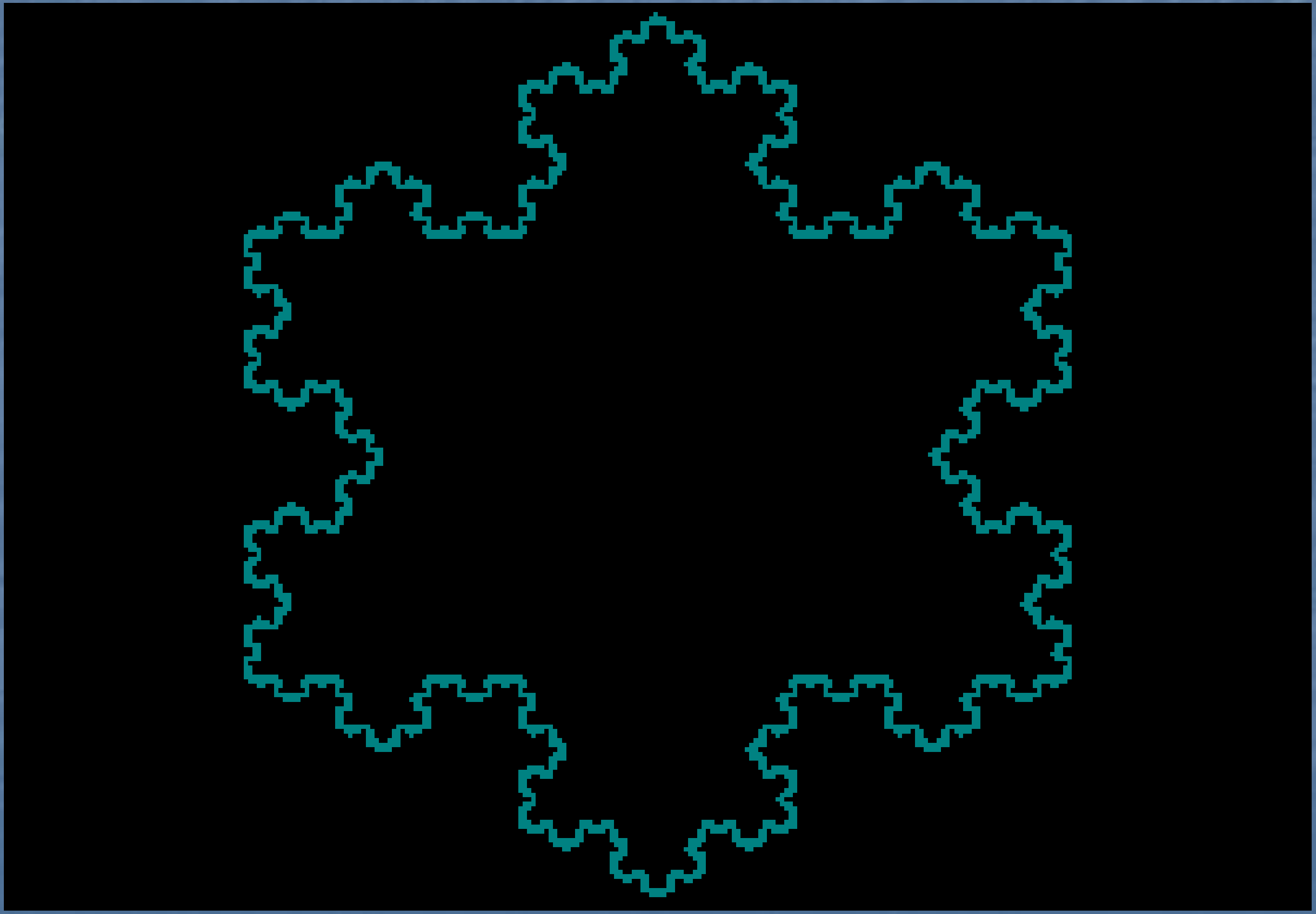
Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html

Landscape wrapped around a sphere (computer generated)



Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html

Snowflakes



Internet site: http://www.kcsd.k12.pa.us/~projects/fractal/pics.html

Conclusion

- Fractals are all around us.
- Mathematicians have developed and are continuing to develop equations to generate these fractals.
- Maybe the stock market and weather will be even more predictable in the future.