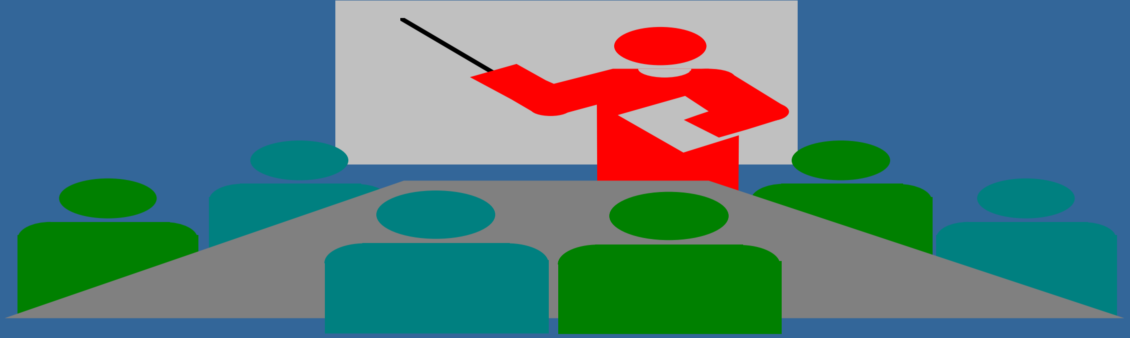

An AIDS Epidemic Model

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Epidemic - a contagious disease that affects an excessive number of people at a time

Past Epidemics

- “Plague of Justinian” 541 A.D.
- Bubonic Plague 1338
- Influenza 1918
- Polio early 1900’s

Simplest Mathematical Model

- **t = time**
- **R(t) = # of infected people at t**

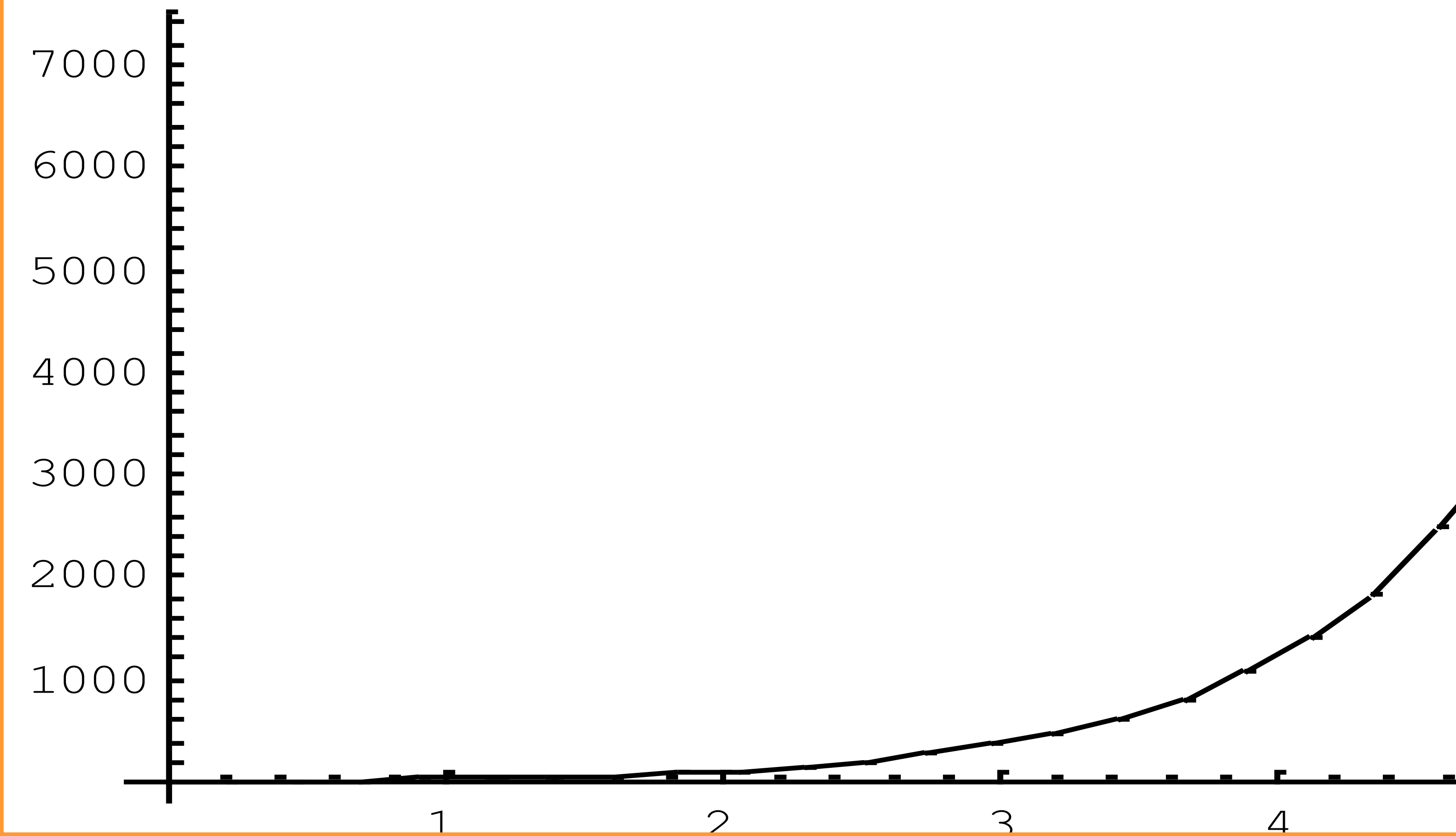
$$\frac{dR(t)}{dt} = kR(t)$$

- **k = constant of proportionality**

$$R(t) = R_0 e^{kt}$$

R_0 = number of infected people at time zero

Exponential Growth



Logistic Model

- **t = time**
- **R(t) = # of infected people at t**
- **N = total people in population**
- **Susceptible = N-R(t)**

$$\frac{dR(t)}{dt} = kR(t)(N - R(t))$$

Formula for Logistic Growth

$$R(t) = \frac{NR_0}{R_0 + (N - R_0)e^{-kNt}}$$

Two important facts about the behavior of R(t)

Small t:

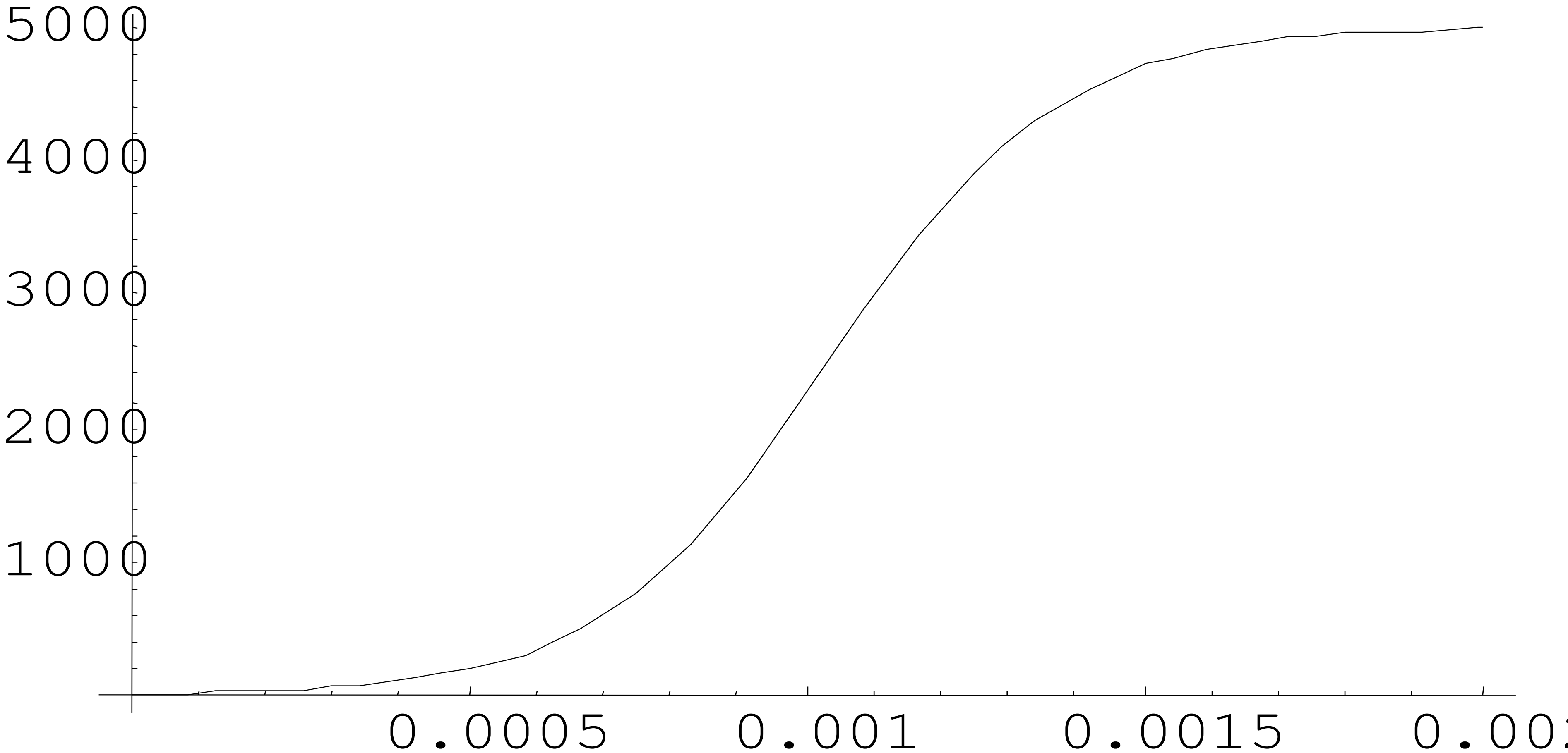
$$R(t) \approx R_0 e^{kNt}$$

For small t, logistic growth looks like exponential growth.

Large t:

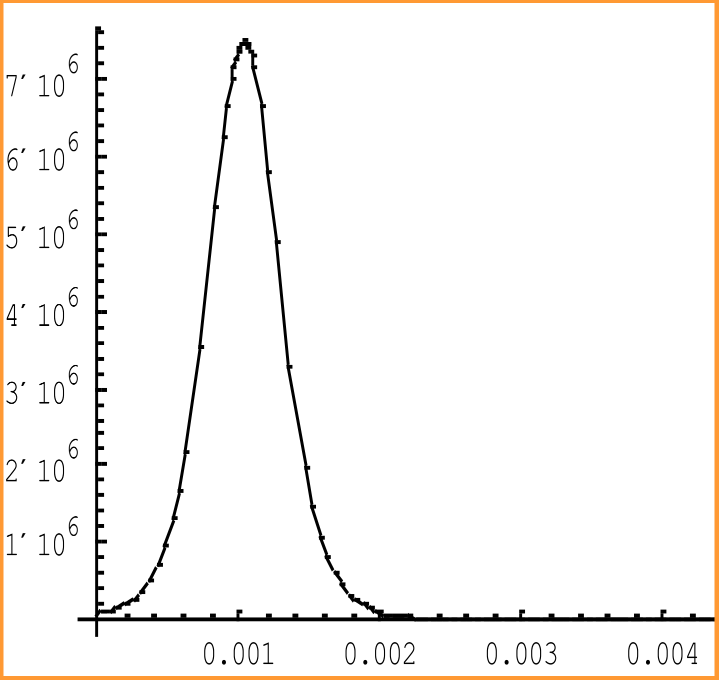
$$\lim_{t \to \infty} R(t) = N$$

Logistic Curve



Epidemic Curve

The growth of any epidemic at time t



Logistic Model Assumptions

- R(t) is assumed to be a continuous function.
- We assumed that the growth rate is proportional to the product of the numbers of infecteds and susceptibles.
- Infecteds and susceptibles are the only two categories of people.
- A newly infected person automatically develops the epidemic.
- Any person can infect any other person.

Facts about AIDS

- AIDS is the fifth leading cause of death.
- Results from an HIV infection
- needle-sharing, blood transfusions, and sexual contact
- Sexual contact results in largest percentage of AIDS cases
- Latency period: 2-18 years

Saturation Wave Model

- **Six Steps**

  -Latency Period

  -Formula for Derivative of A(t)

  -Heterogeneous Behavior

  -Growth in Single Risk Group

  -Saturation Wave and HIV Infection

  -Cubic Growth of AIDS

Step 1: Latency Period

- **L(t) = probability density function for the latency period**

$$\int_{\tau}^{\tau+\Delta\tau} L(t)\,dt$$

Step 2

$$A(t+\Delta\tau) - A(t) \approx \sum_{i=1}^{n} [H(t-\tau_i) - H(t-\tau_i - \Delta\tau)] L(\tau_i)\Delta\tau$$

$$A'(t) = \int_{2}^{t} H'(t-\tau) L(\tau) d\tau$$

Step 3: Heterogeneous Behavior

- **r = risk factor**
- **N(r)= # of individuals with risk, r**

$$N(r) \approx \frac{N_0}{r^3}$$

Step 4: Growth in single risk group

$\lambda =$ Proportionality constant

$H_r(t) =$ # of individuals with risk r that have the HIV infection

$$H_r(t) = H_r(0)e^{\lambda rt}$$

$H_r(0)$ = # of infected individuals when we start measuring time

When will the entire group be infected?

$$N(r) = H_r(0)e^{\lambda rt}$$

At what time will this occur?

$$t = \frac{1}{\lambda r} \ln \frac{N(r)}{H_r(0)}$$

Step 5: Saturation Wave and HIV Infection

$$\int_{r_*}^{\infty} N(r)\,dr = \int_{r_*}^{\infty} \frac{N_0}{r^3}\,dr = \frac{N_0}{2r_*^2}$$

r_* = group that just reached saturation

Step 6: Cubic Growth of AIDS

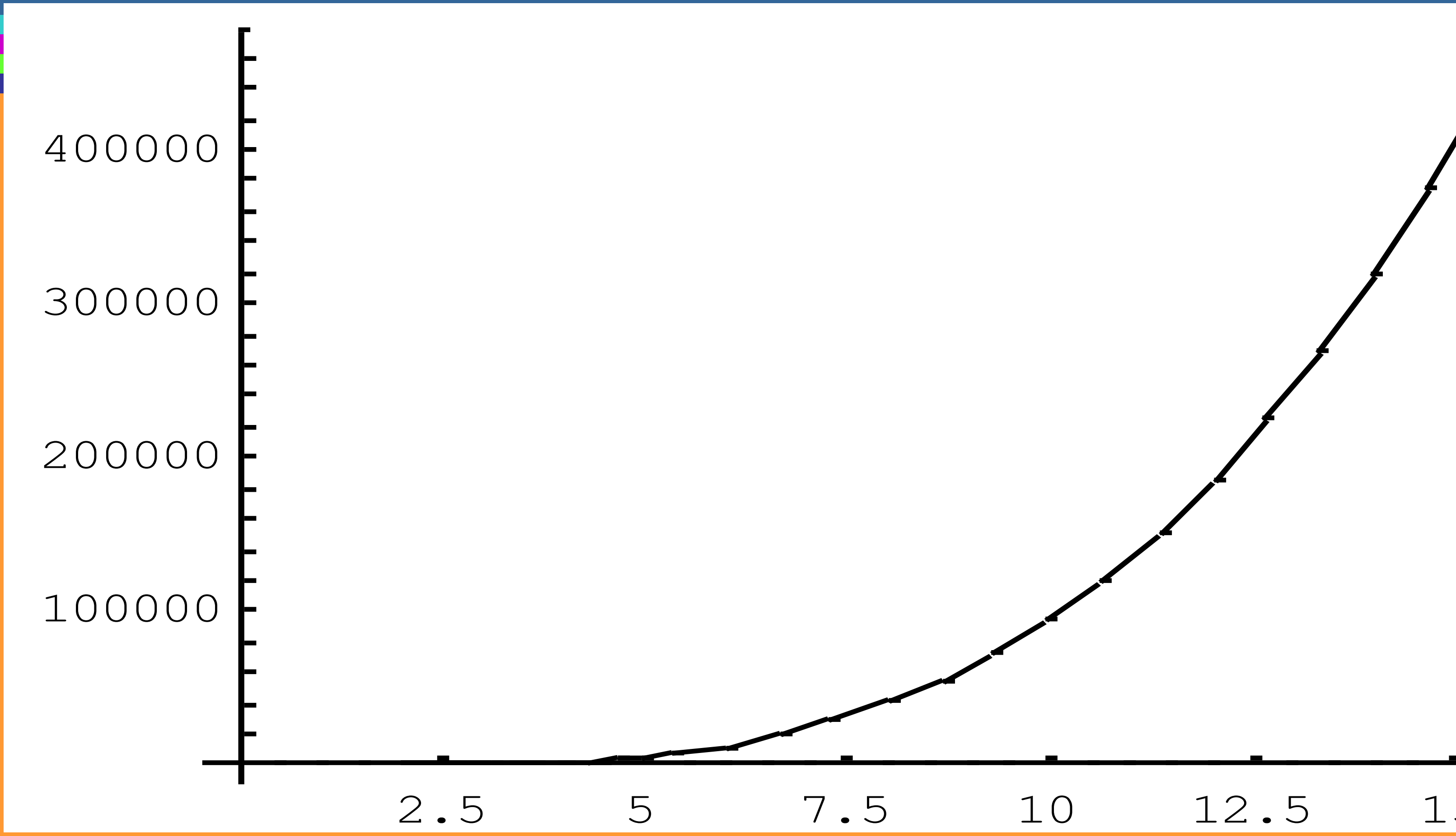
$$L(t) = .0625, \quad 2 \le t \le 18$$

$$A'(t) = .0625 \cdot K(t-2)^2$$

$$A(t) = .0625K \cdot \frac{1}{3}(t-2)^3$$

The cumulative number of AIDS cases is a cubic function of time.

Cubic Growth of AIDS



An Estimate of AIDS

The year 1998:
900,307

The year 1999:
1,068,710

The End