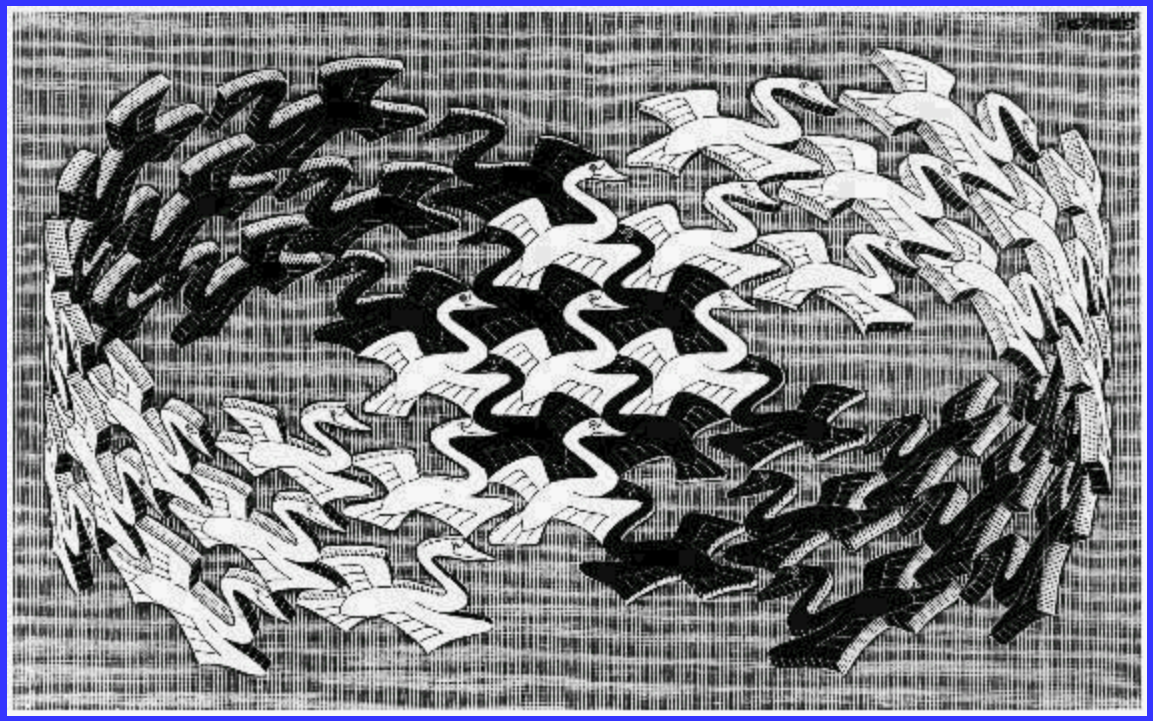


# MATH AND ART

The influence of math in the art of M.C. Escher

by Melissa Culpepper

# OUTLINE

- I. The Life of M.C. Escher
- II. Infinity
  - 1.Endless cycles
    - A. Mobius strip
    - B. Perspective

- 2. Regular Divisions of the Plane
  - A. Symmetry
    - 1. Translation
    - 2. Rotation
    - 3. Reflection
    - 4. Glide reflection

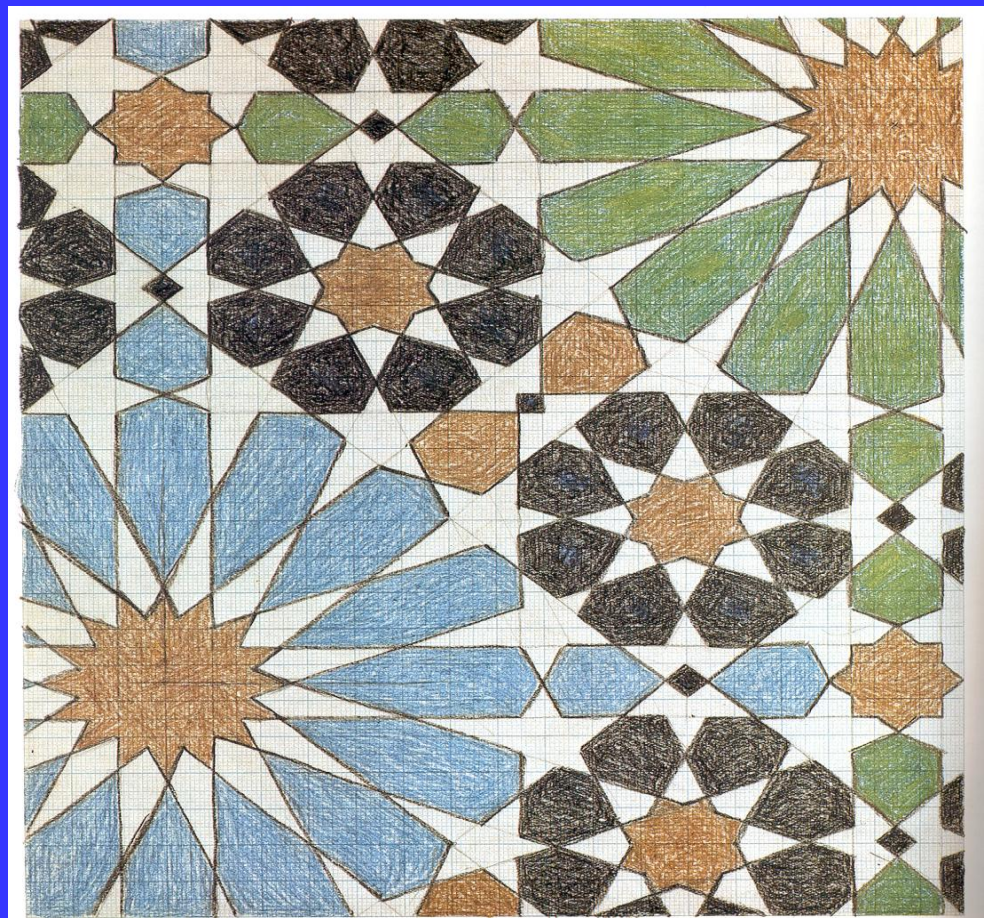
- 3. Limits
  - A. Poincare's model for hyperbolic geometry

# His Life

- Maurits Cornelis Escher
- June 17, 1898

- Failed all classes except drawing
- Failed all finals

- 1919: Alhambra in Granada, Spain
  - His inspiration for tessellations
    - no plant or animal form
    - "great complexity and geometric artistry"



- 1935: 2nd visit to Alhambra
  - still intrigued him
  - no longer concerned with *expression of his observations, but with the construction of the images of his mind--* **TURNING POINT OF HIS LIFE**

- regular division of the plane
- limitless space
- rings and spirals in space
- mirror images
- inversion
- polyhedrons
- relativities
- conflict between flat and spatial
- impossible constructions

# *Why his work took a turn….*

- Landscapes no longer of interest
- Compelled him to withdraw from the directness of his surroundings and bring inner visions into being
- "Wonder of the laws of nature"

- "Although I am absolutely without training or knowledge in the exact sciences, I often seem to have more in common with mathematicians than with my fellow artists."

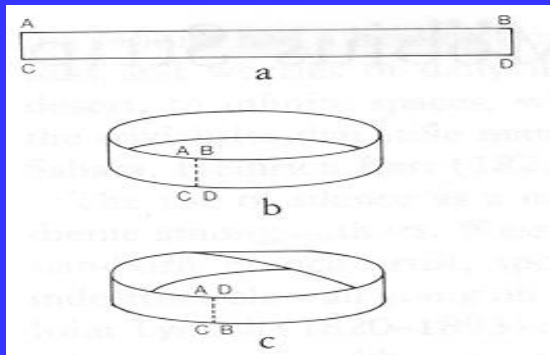
# *Infinity*

- Endless Cycles
  - Fascination with rhythm, regularity, and periodicity
  - Perpetual motion

"I cannot help mocking …great fun deliberately to confuse two and three dimensions, the plane and space, or to poke fun at gravity"

# Mobius Strip

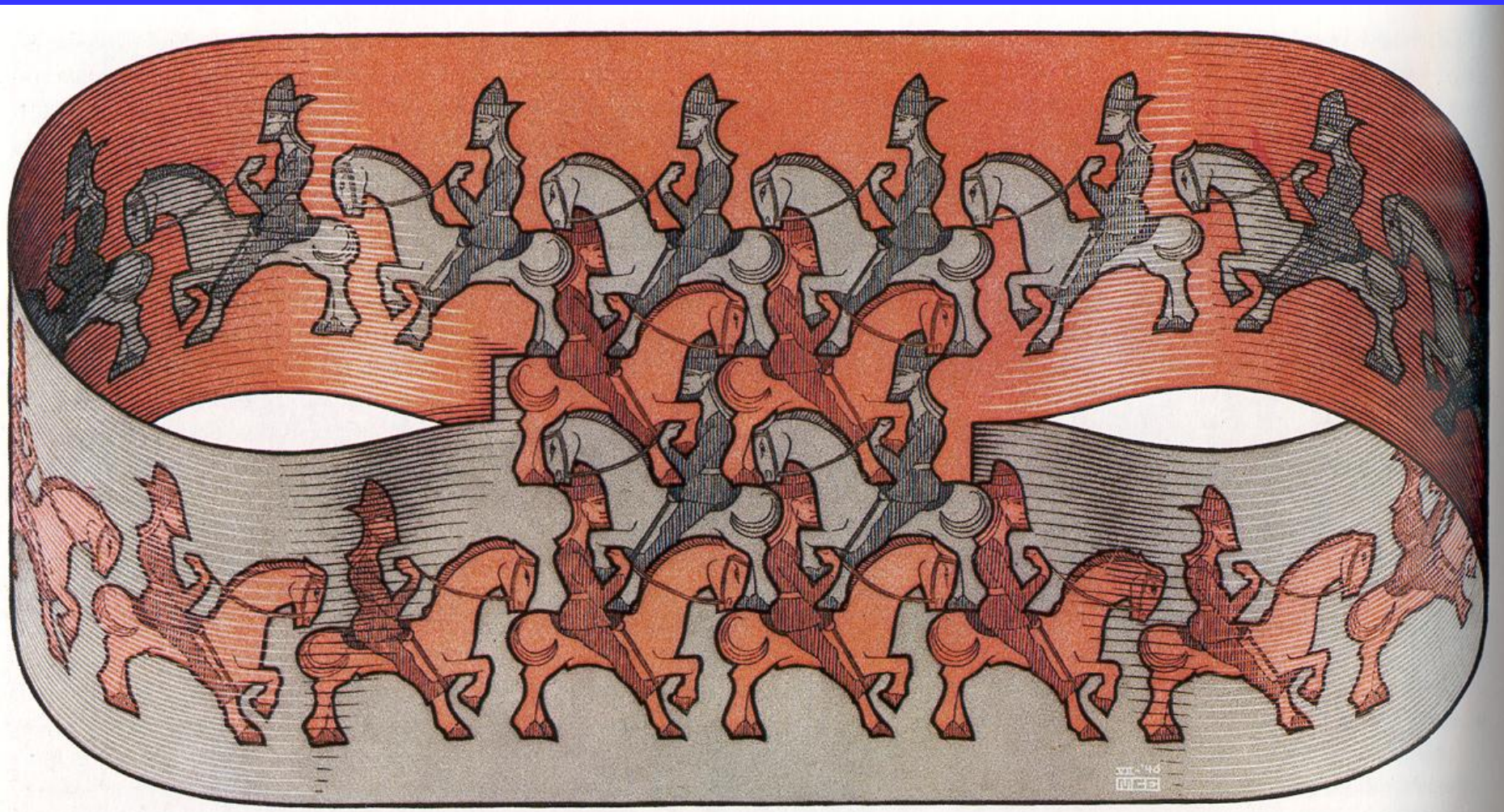
- August Ferdinand Mobius (1790-1868)
  - "unilateral polyhedra"
- Topology
  - study of those properties of a surface which remain invariant when the surface undergoes a continuous deformation



# *Mobius Strip*

- Continuous path without ever penetrating the surface or going over its edge
- neither an "inside" nor an "outside"
- encounter with an English mathematician
- 3 works

# Grotesque creations team with life



# Perspective

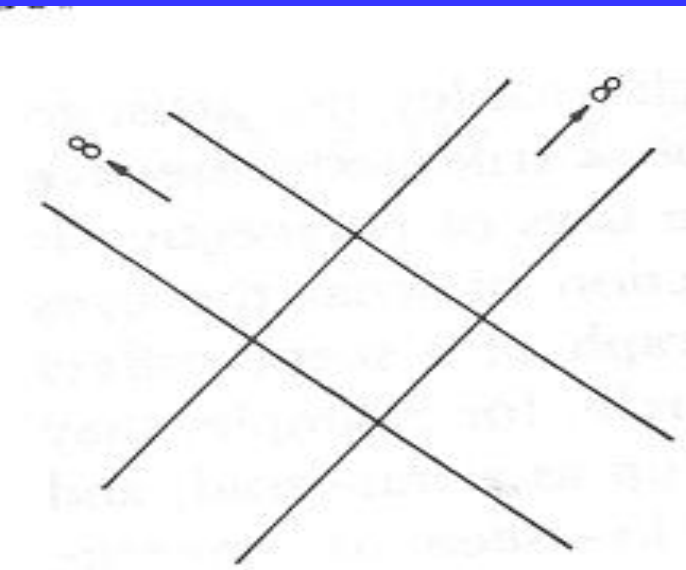
- Renaissance - depth missing
- Perspective- a graphical scheme which enables the artist to put on canvas the scene in front of him in a truly objective manner
  - photo-might differ from the actual scene

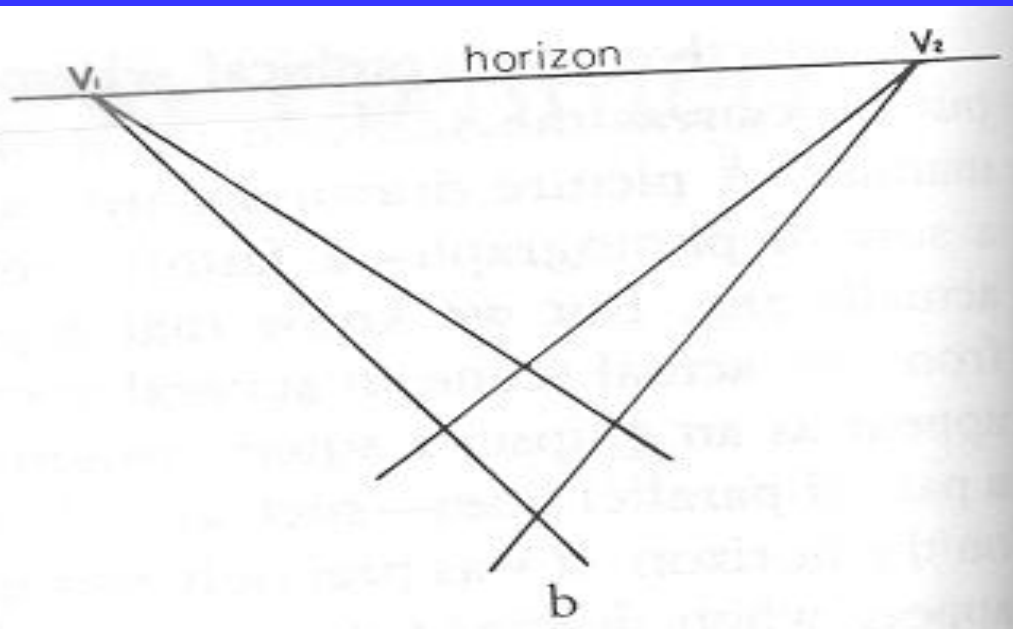
# How does an object appear when depicted on canvas?

- 16th century-Perspective Geometry
  - deals with those properties that remain unchanged under a projection-what the eye sees

# *Points and Lines at Infinity*

- Euclidean parallel's
- Projective parallel's-converge to a point



- Parallel lines and ordinary lines
  - any 2 lines in the plane meet at one and only one point
    - elegance of math and art

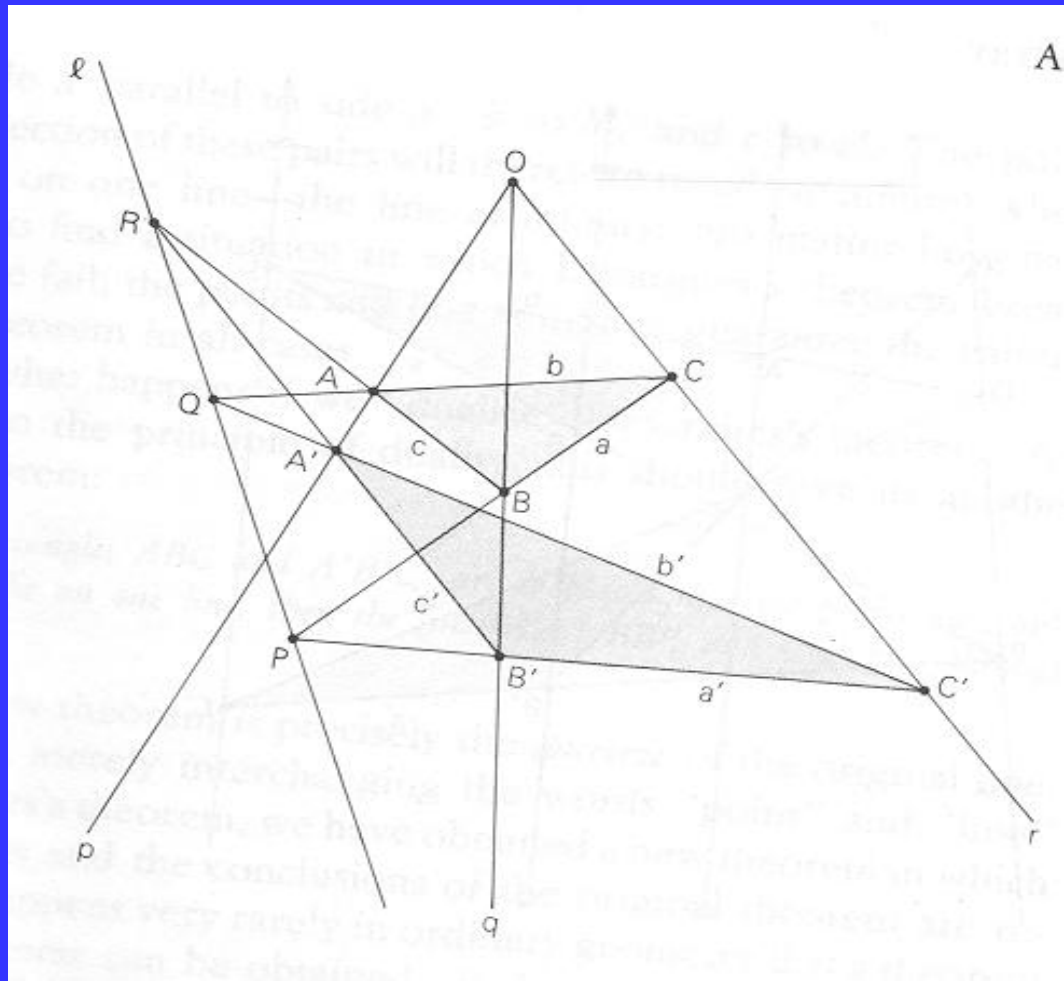
- Ordinary points and points at infinity
  - ordinary points determine a position
  - points at infinity determine a direction

  Principle of Duality between points and lines

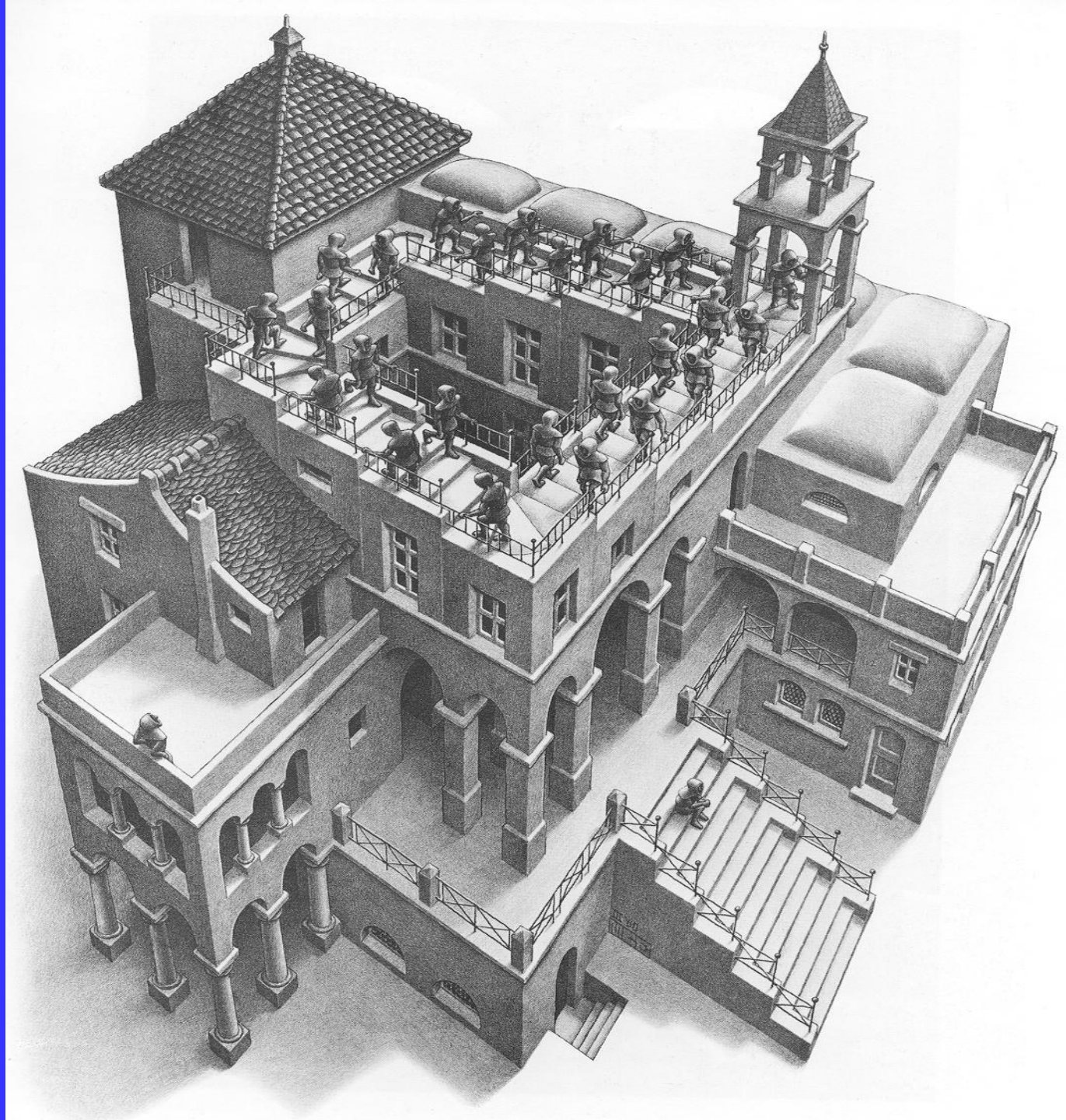
Triangles ABC and A′B′C′ are projections of each other with O as the point of projection.

Example of a 2-D geometric configuration that is easier to prove when viewed from a 3-D perspective.

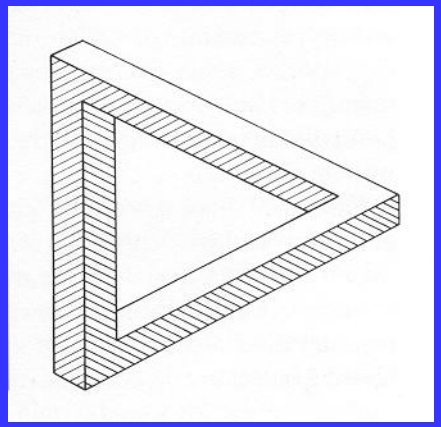
Escher's style

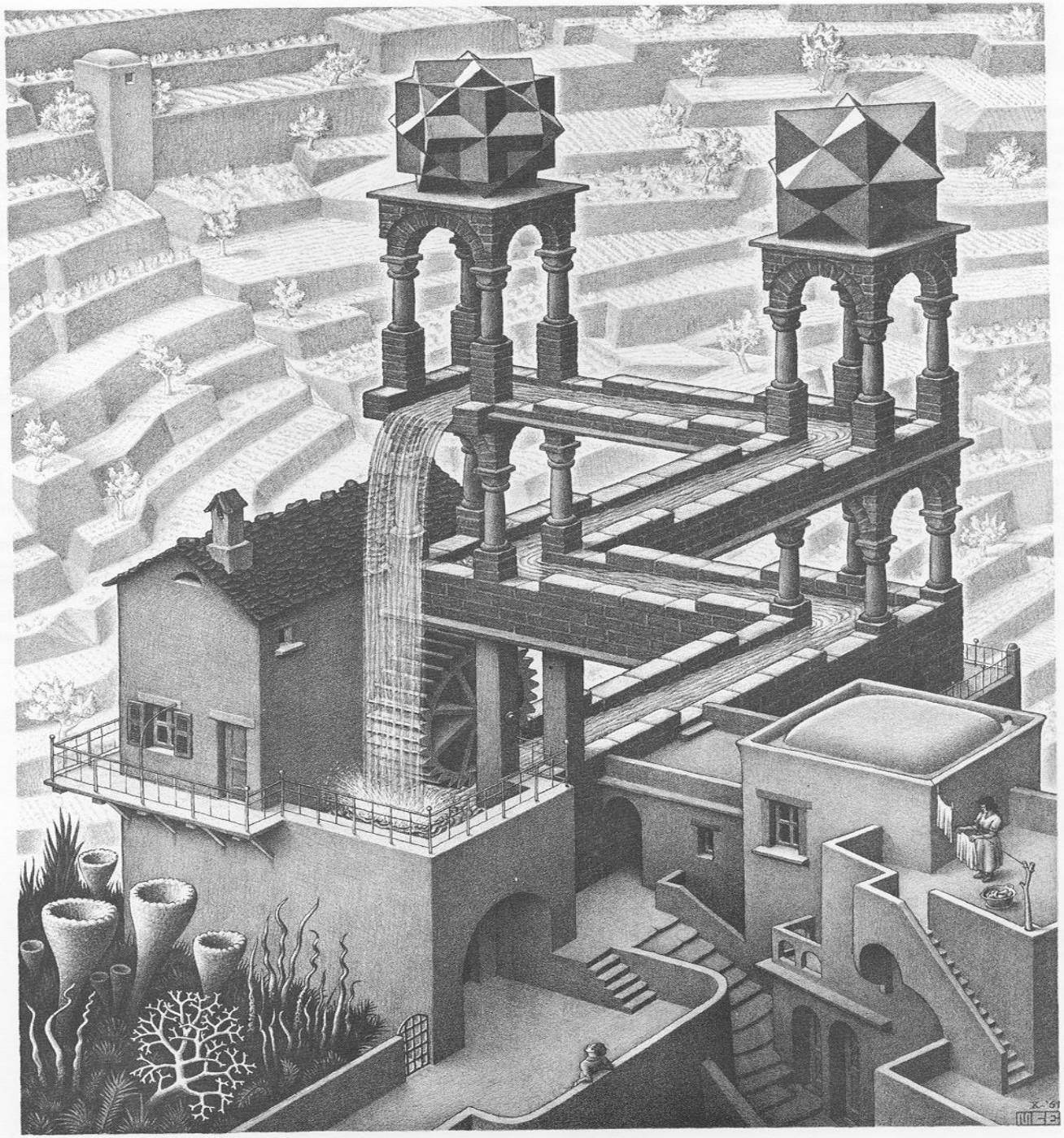


"Yes, yes, we climb up and up, we imagine we are ascending; every step is about ten inches high, terribly tiring-and where does it get us? Nowhere; we don't get a step farther or higher."



Roger Penrose: Penrose Triangle-3 bars at right angles seem to form a spatial object. Something wrong with the connection.

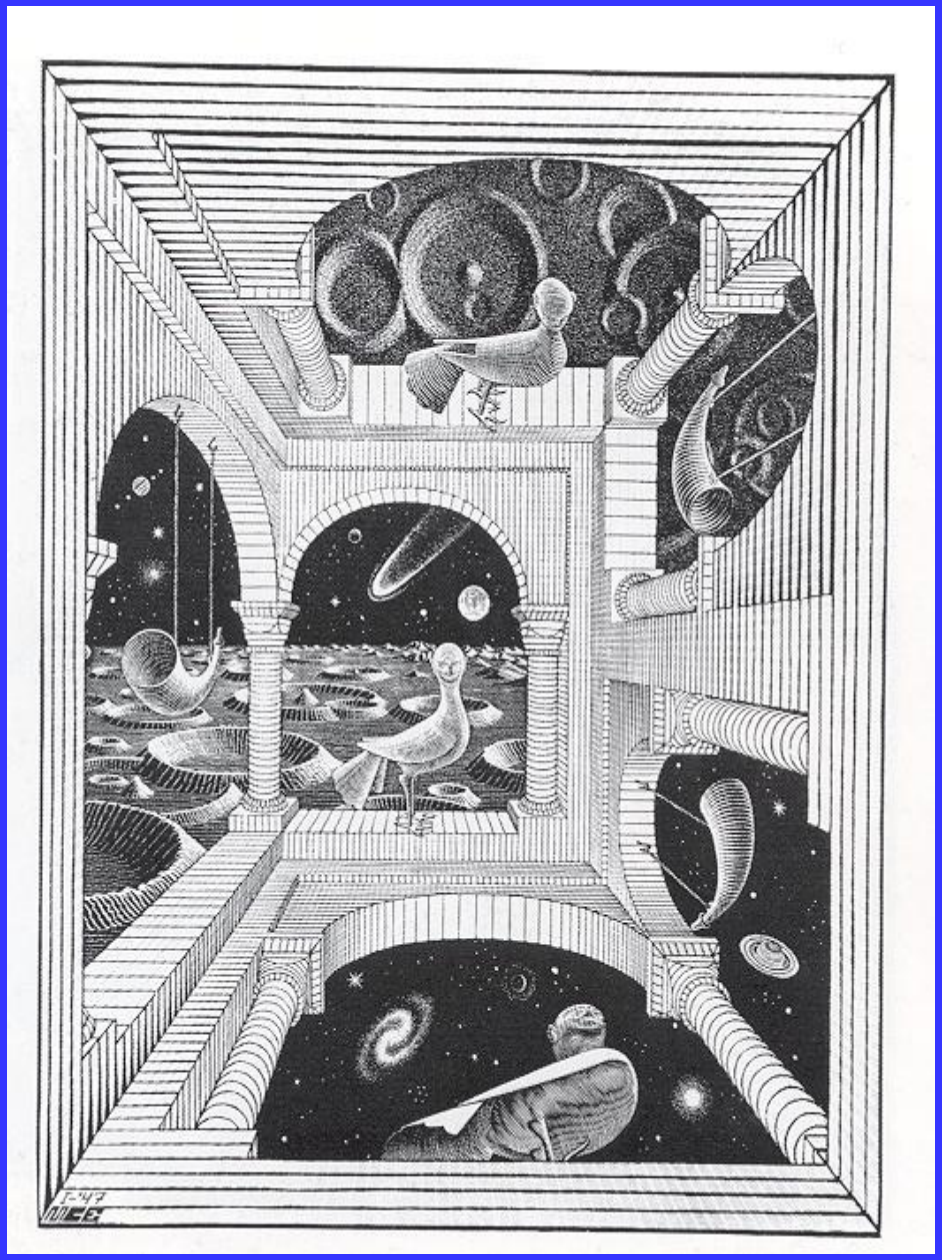




# Zenith and Nadir

- Zenith:point of perspective directly above your head
- Nadir:point of perspective directly below your feet

- Interchangeable



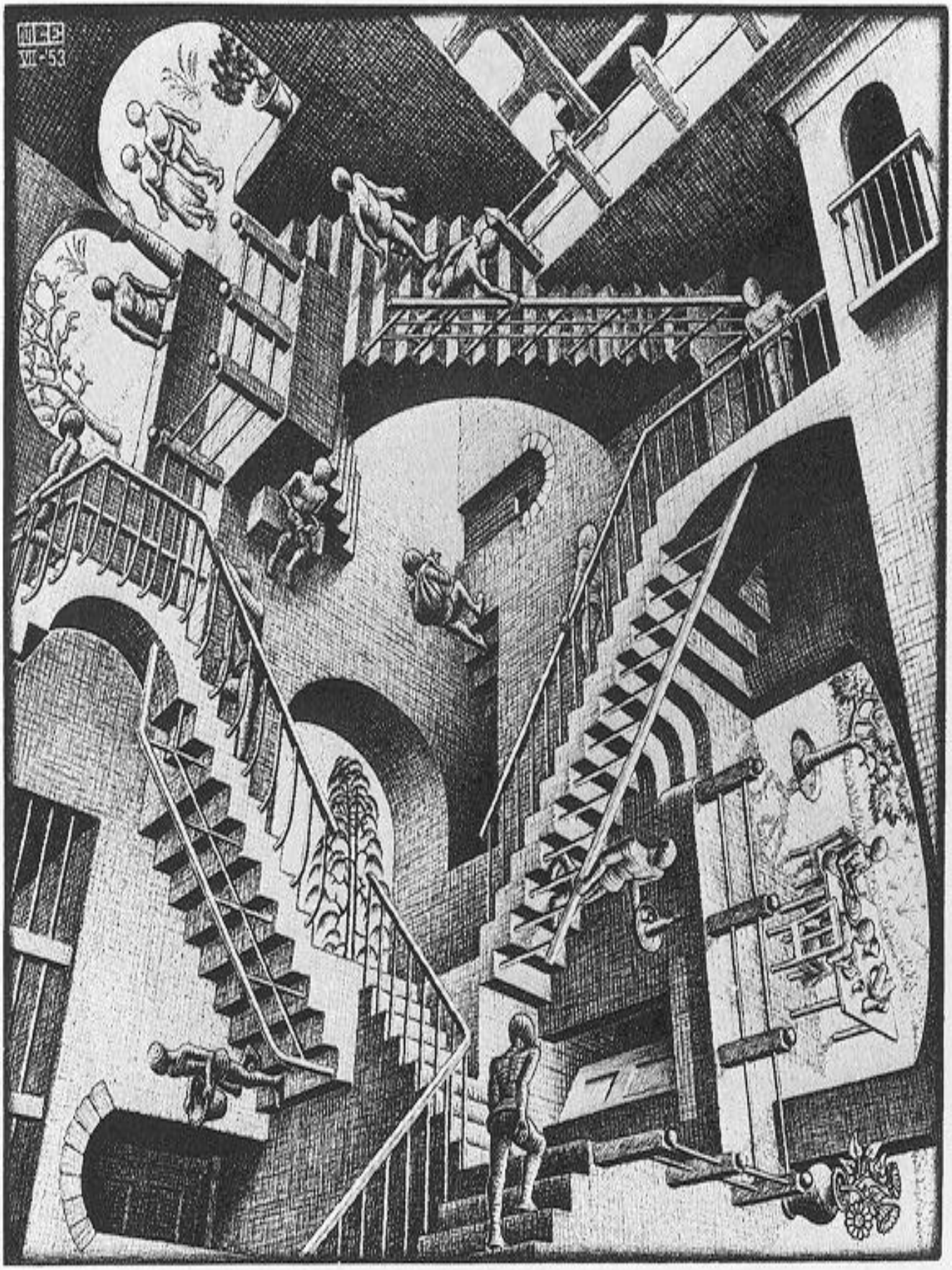
Up

down

left

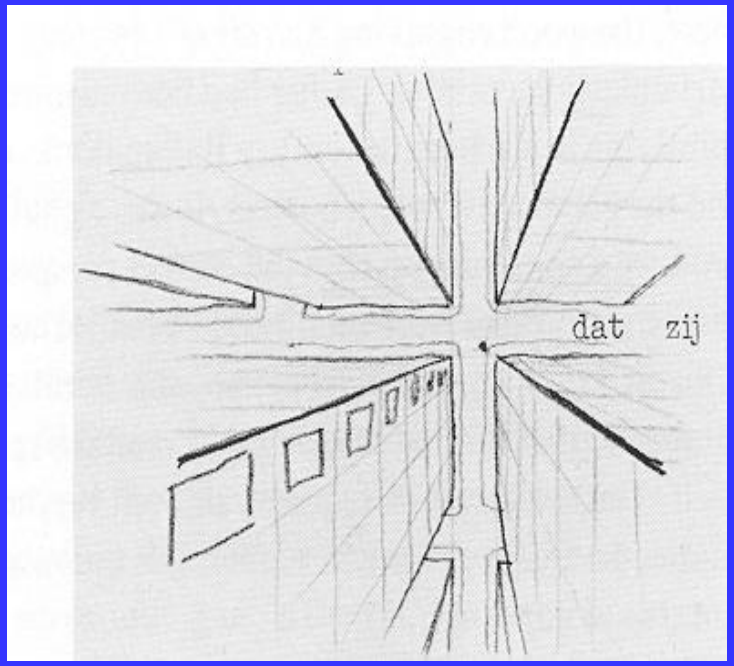
right

which window?????

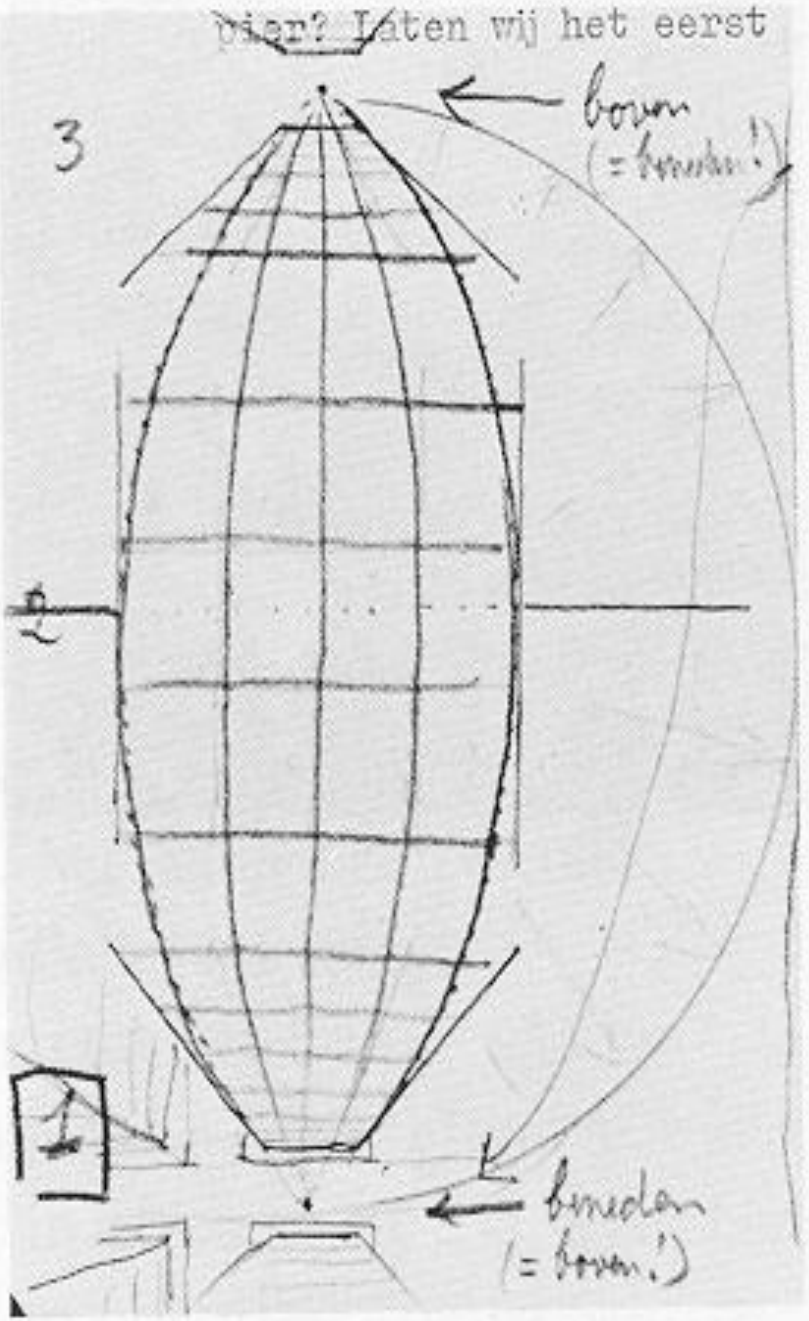

M.C.E.
VII-53

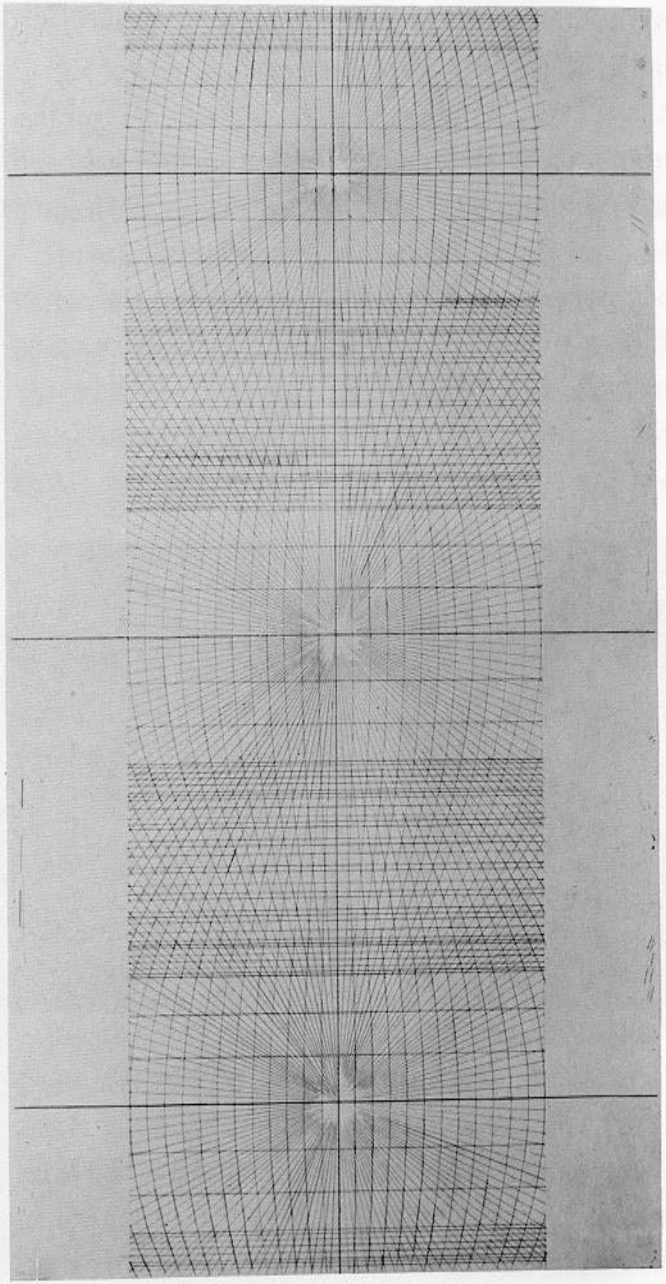
# Curved Perspective Lines

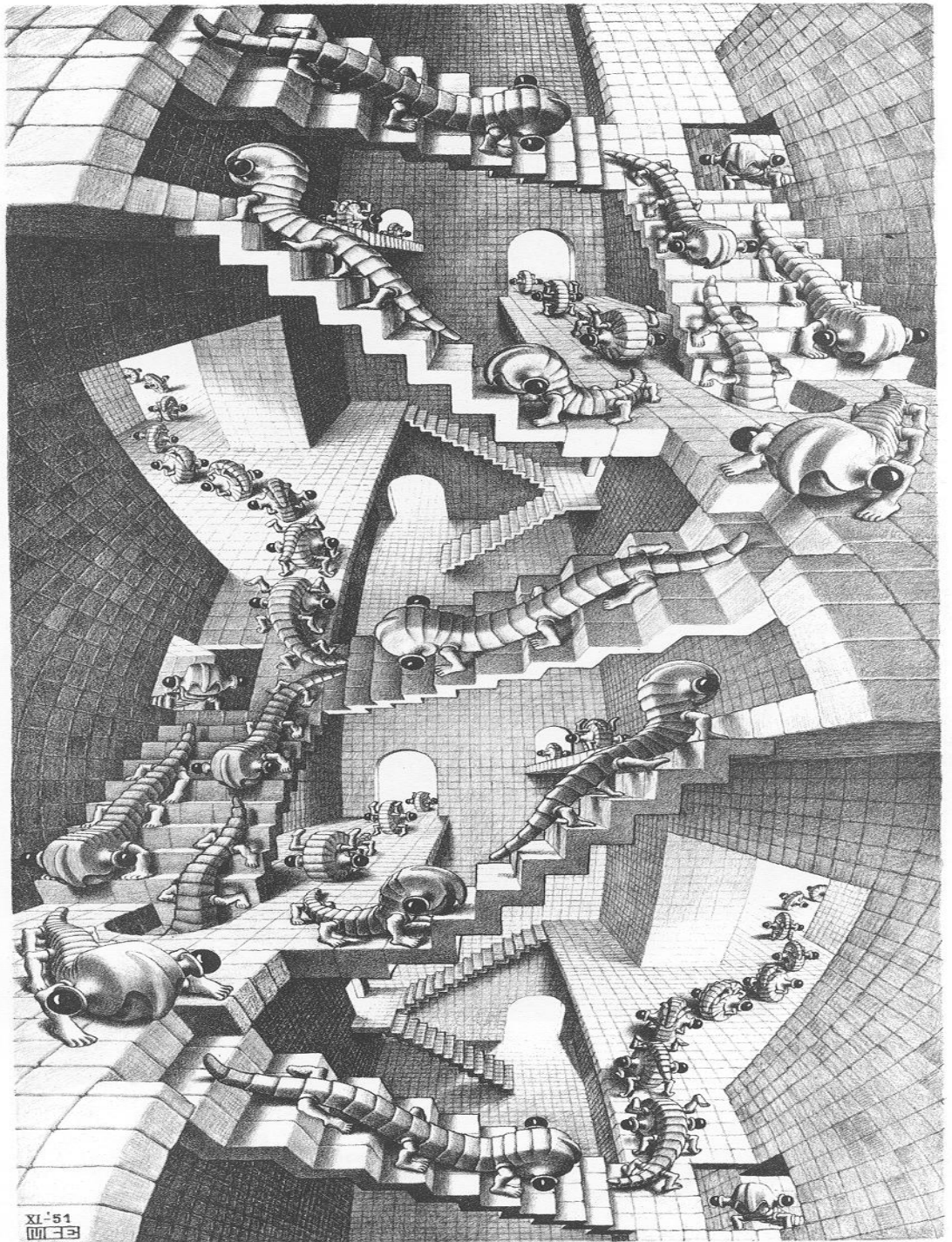
- 1st: Look down
- 2nd: Look at the horizon
- 3rd: Look straight up




- "In order to produce a single continuous picture from these three separate ones, draw a curved line from zenith to nadir, which touch the corresponding lines of the three stages"






XI-'51

# *Regular Division of the Plane*

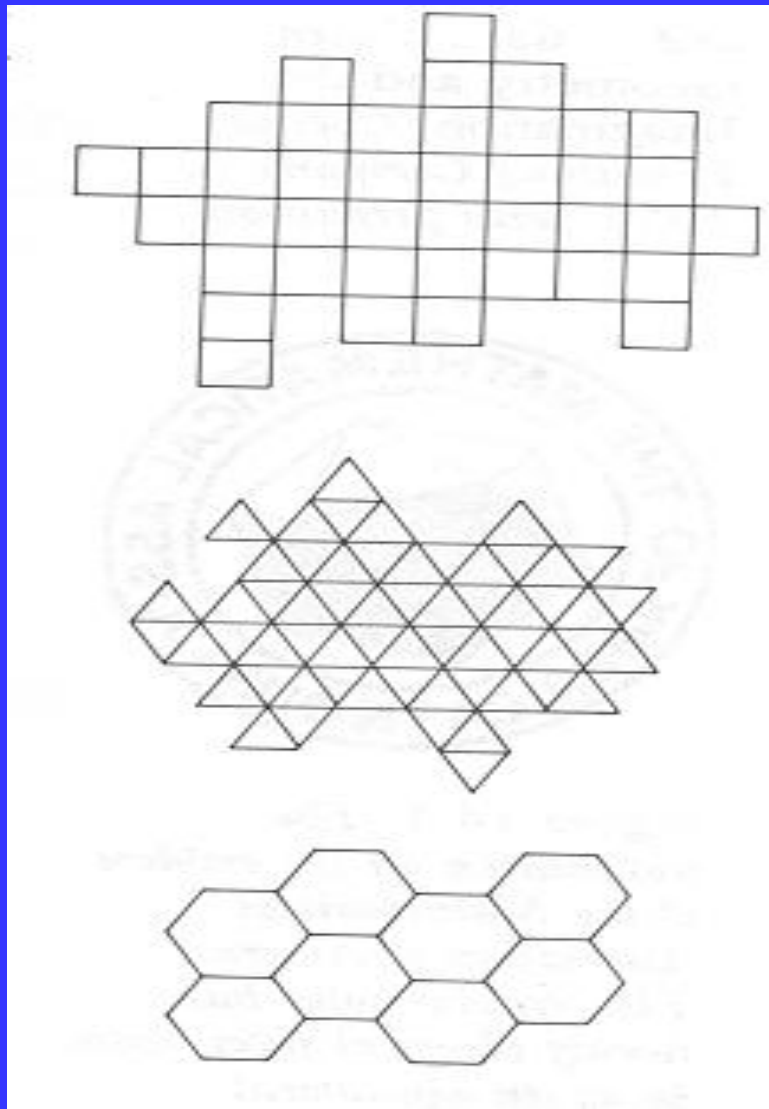
*Tessellations*

- Escher's hallmark
- unlike those of Moors
  - irresistible challenge
  - recognizable objects

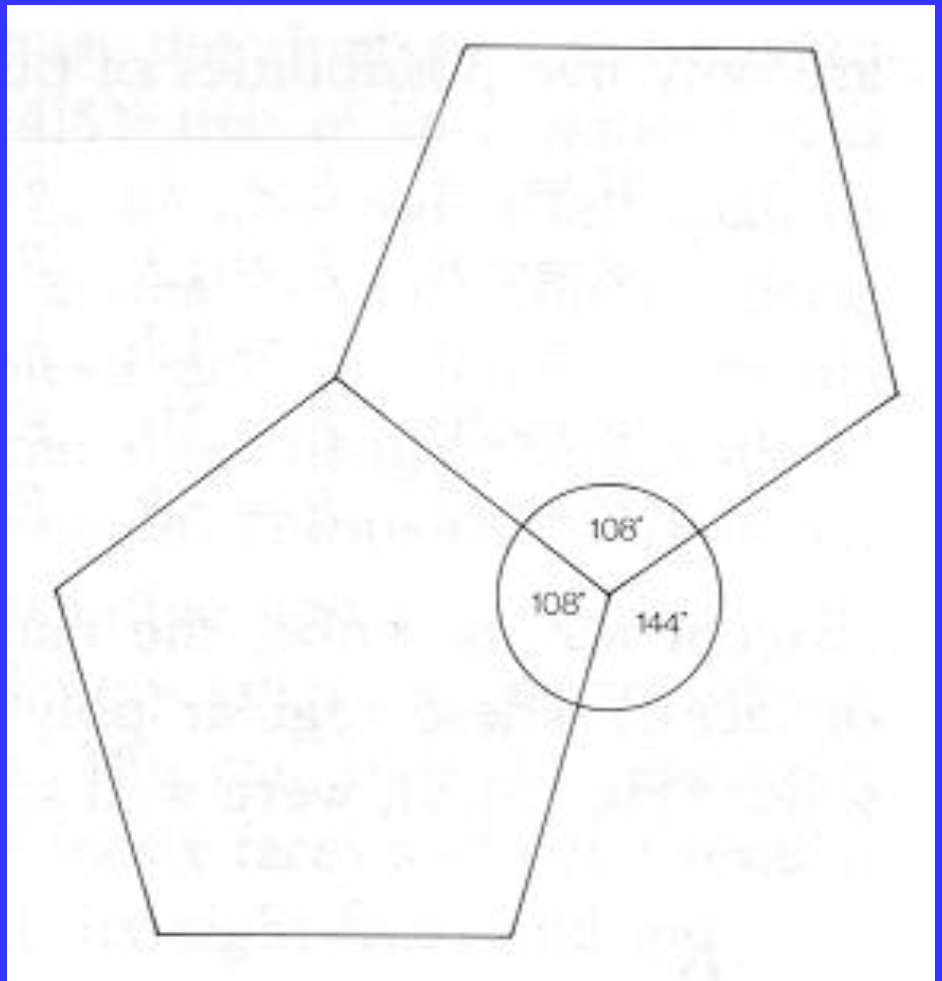
"A plane which should be considered limitless on all sides, can be filled with or divided into similar geometric figures that border each other on all sides without leaving any 'empty spaces'. This can be carried on to infinity according to a limited number of systems."

# 3 Figures that Tessellate

- Square
- Equilateral Triangle
- Hexagon



- Sum of measures of angles around any vertex must equal 360 degrees
- All other n-gons eliminated




# *Crystallography*

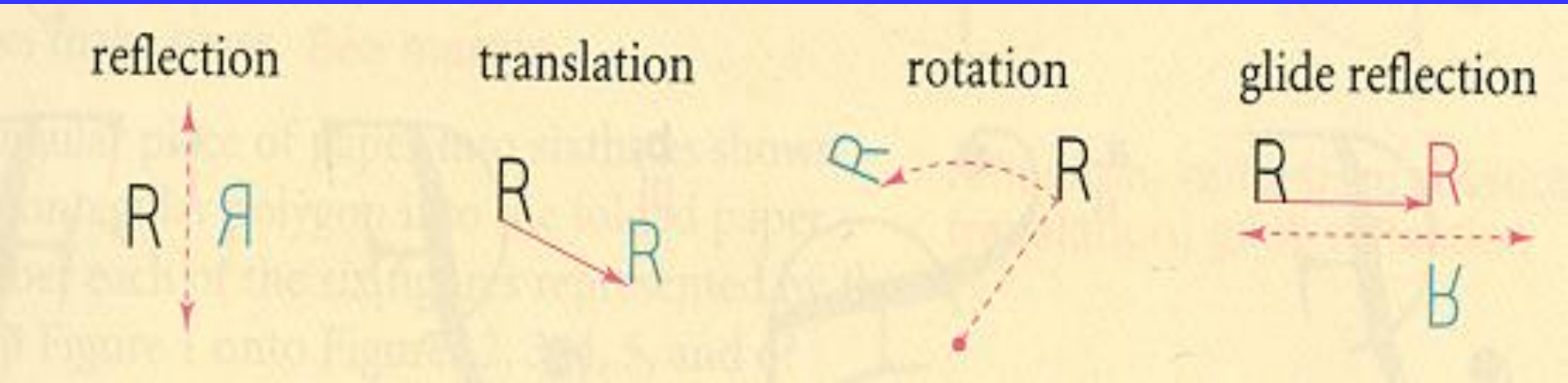
- Spatial regularity
  - unit cells-fill the volume of a crystal

Tessellations
- unit designs-fill the plane

Periodic Lattice
- global order
- isometries

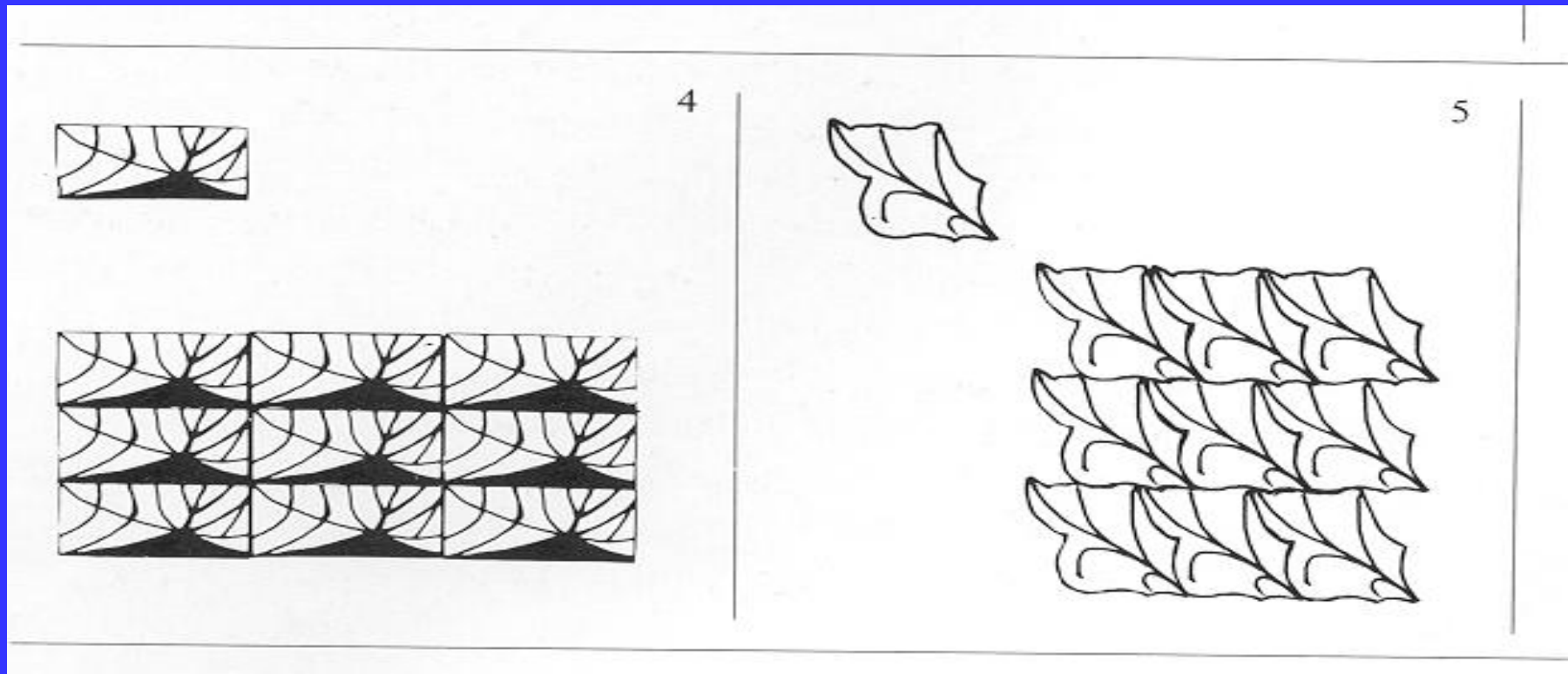
# Tessellations

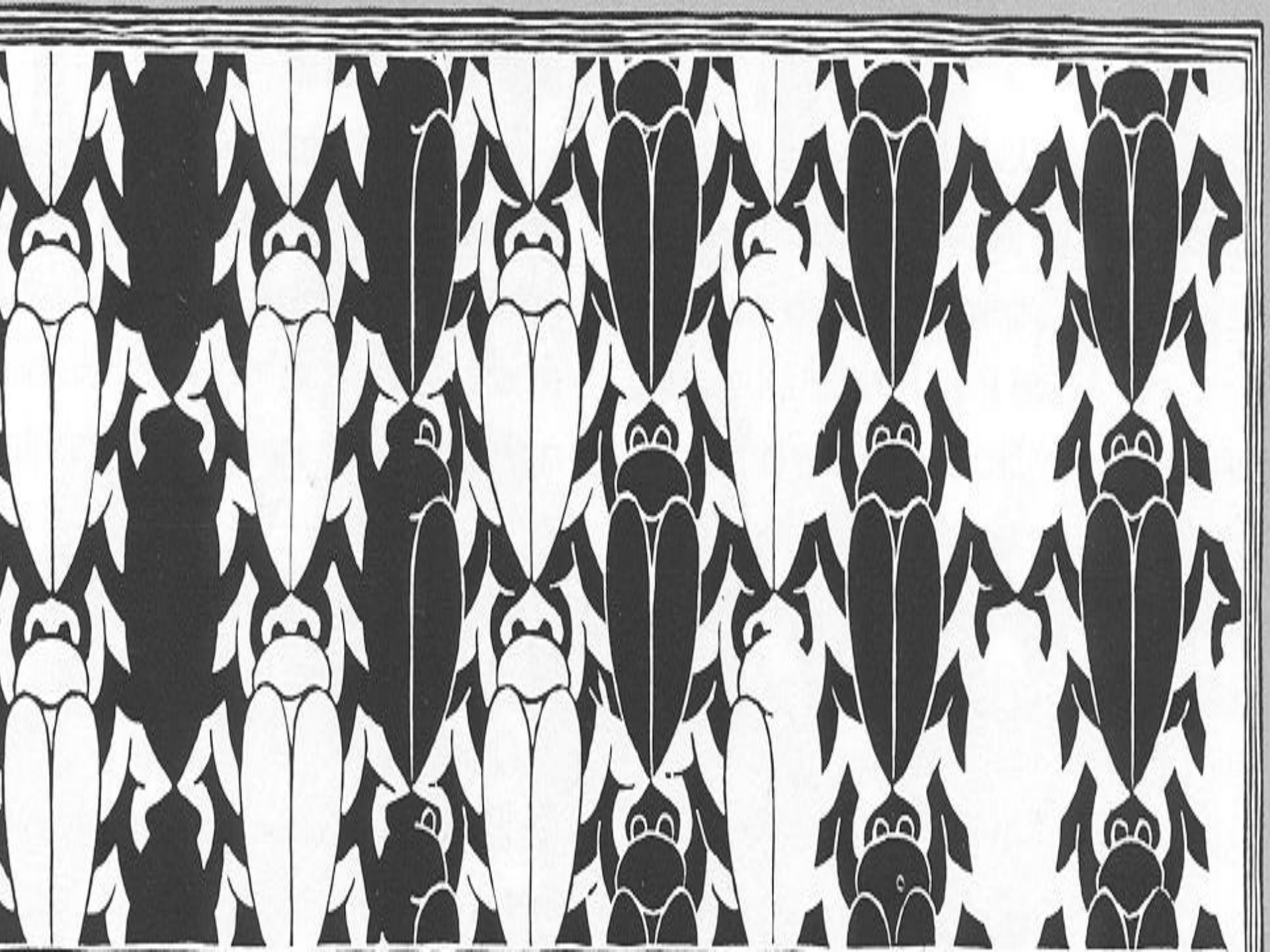
- 4 Basic Transformations
  - translation
  - rotation
  - reflection
  - glide reflection



# Translation

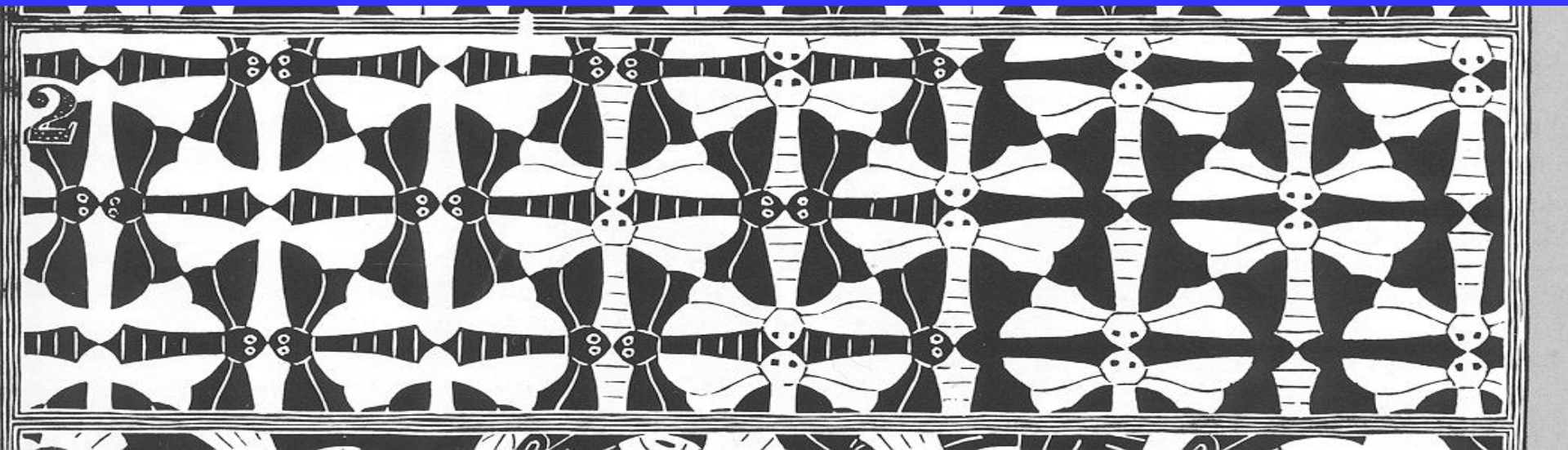
- Sliding motion-moves points same distance and same direction





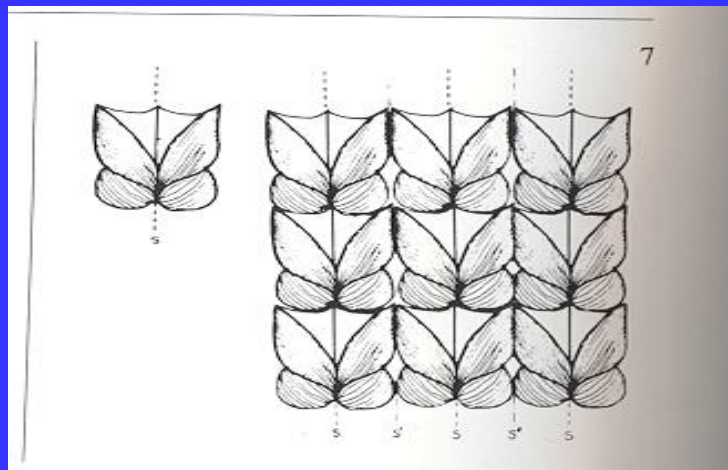
# *Rotation*

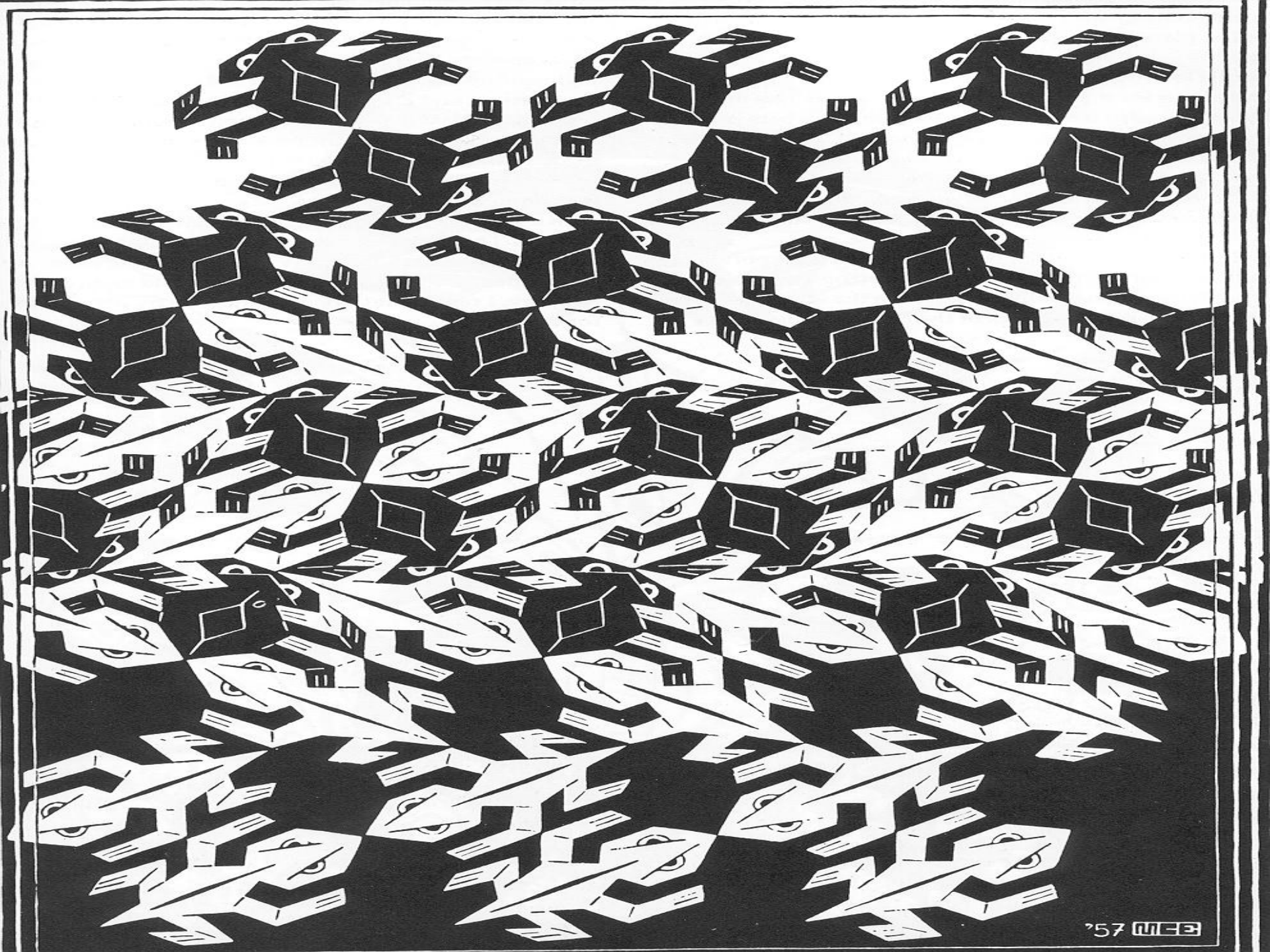
- Center of Rotation
- Angle of Rotation
- Direction of Rotation
  - clockwise or counterclockwise



# Reflection

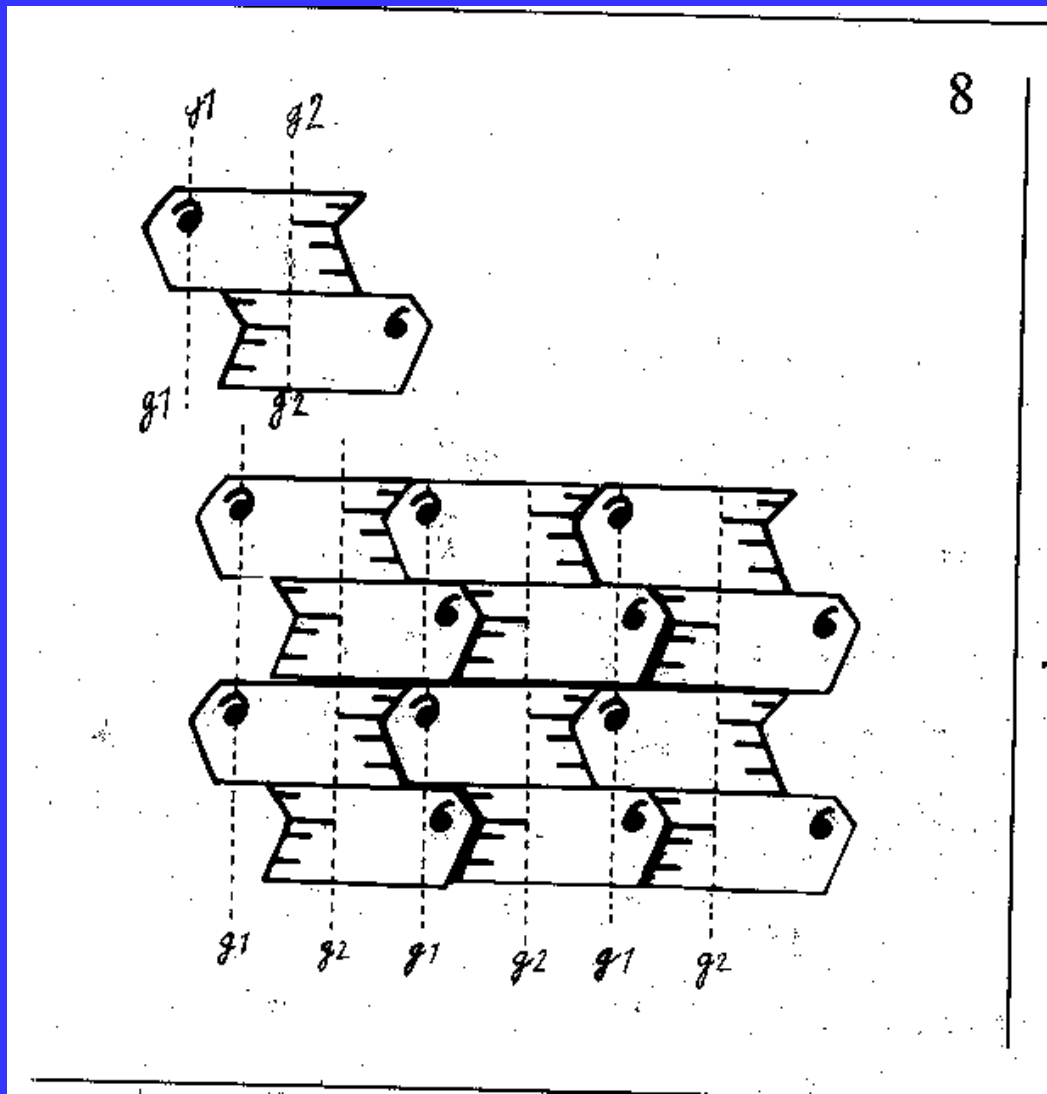
- Line or point to reflect over
  - gives mirror image
  - reverses orientation
  - line of reflection is perpendicular bisector of the segment connecting a point and its image




'57 MCE

# Glide Reflection

- Translation
- Reflection

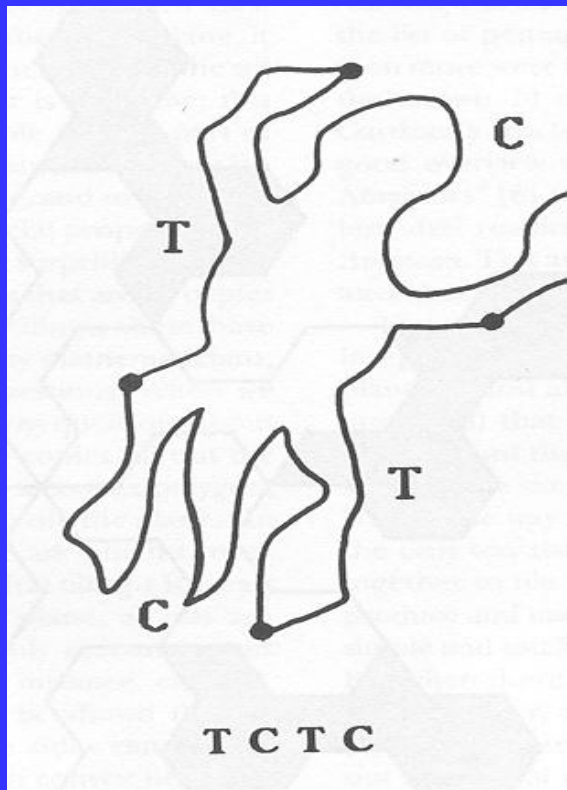


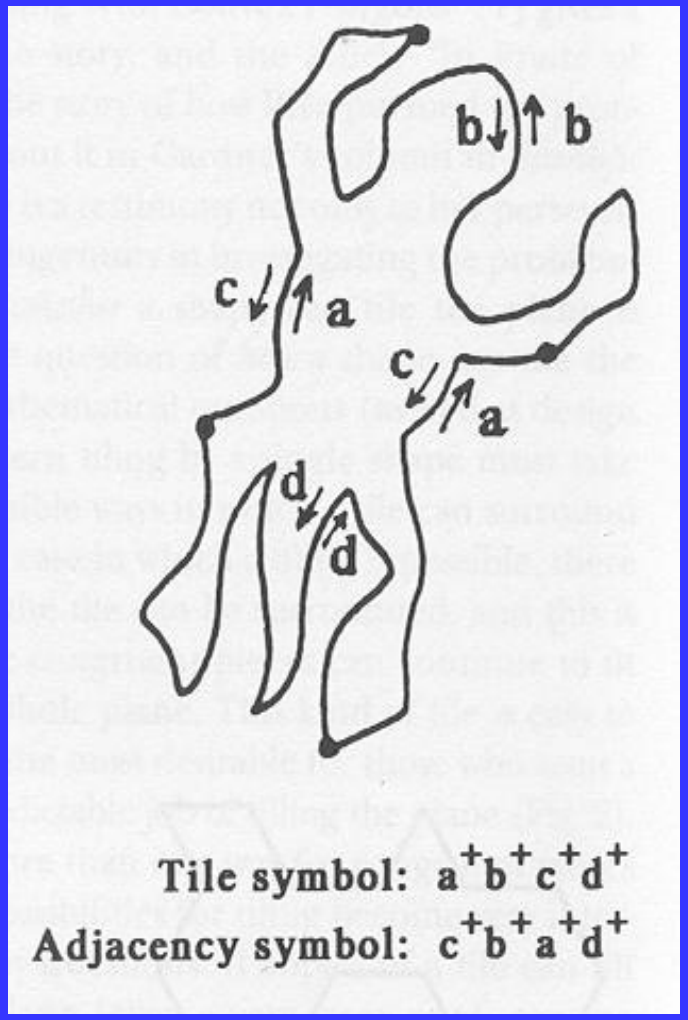

M C E '57

# *Fitting Together*

- Label edges consecutively
- Escher described how each

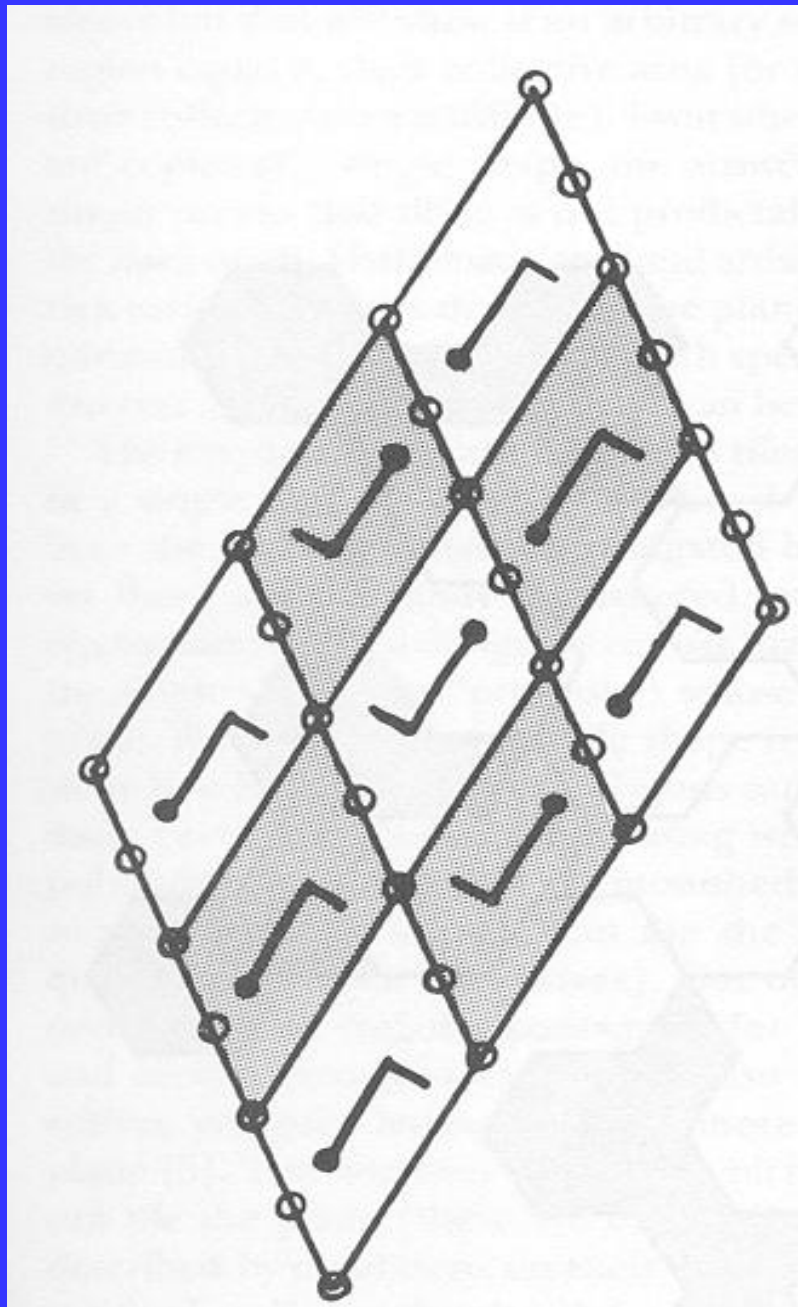
tile was related by
isometries
to its
surrounding tiles

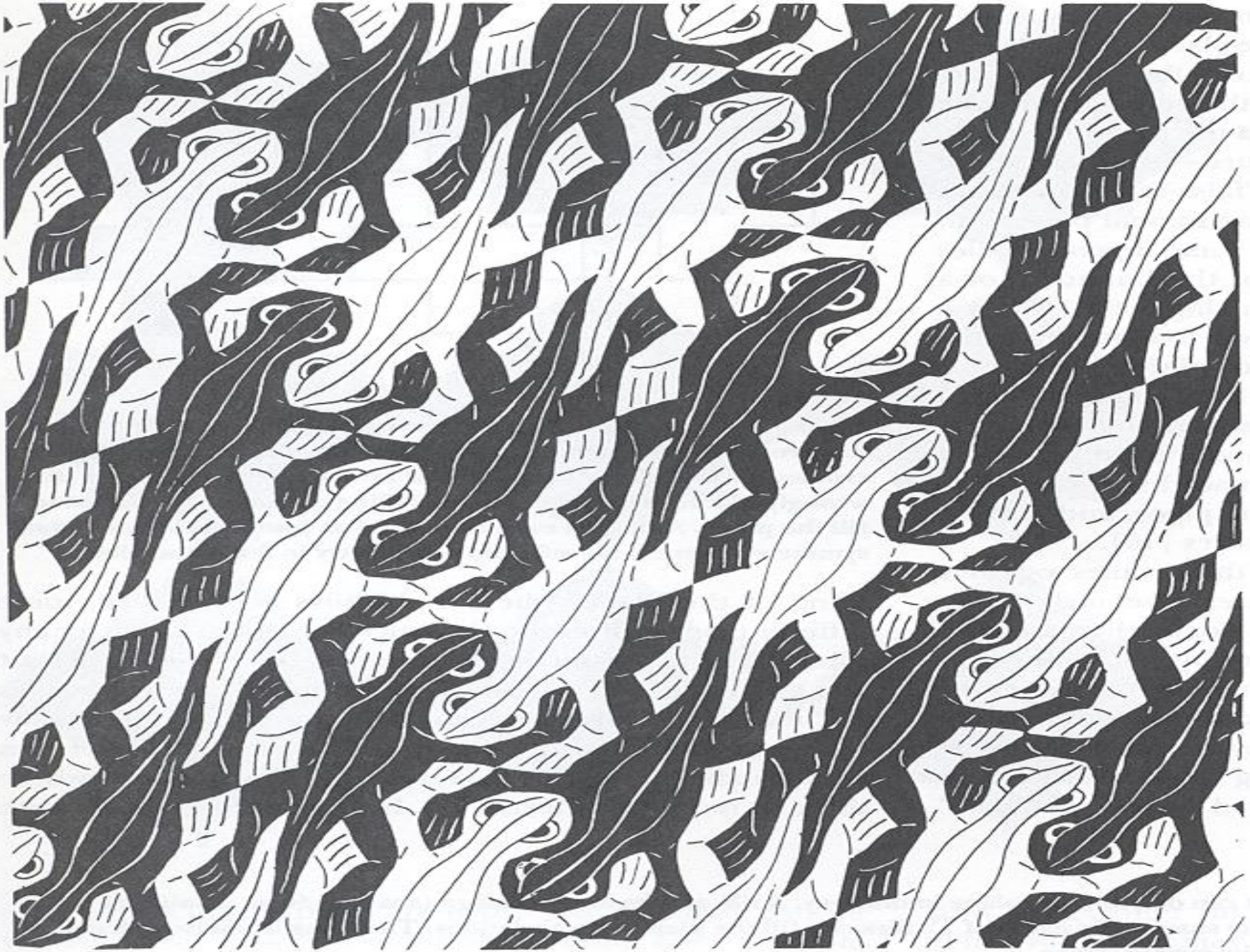


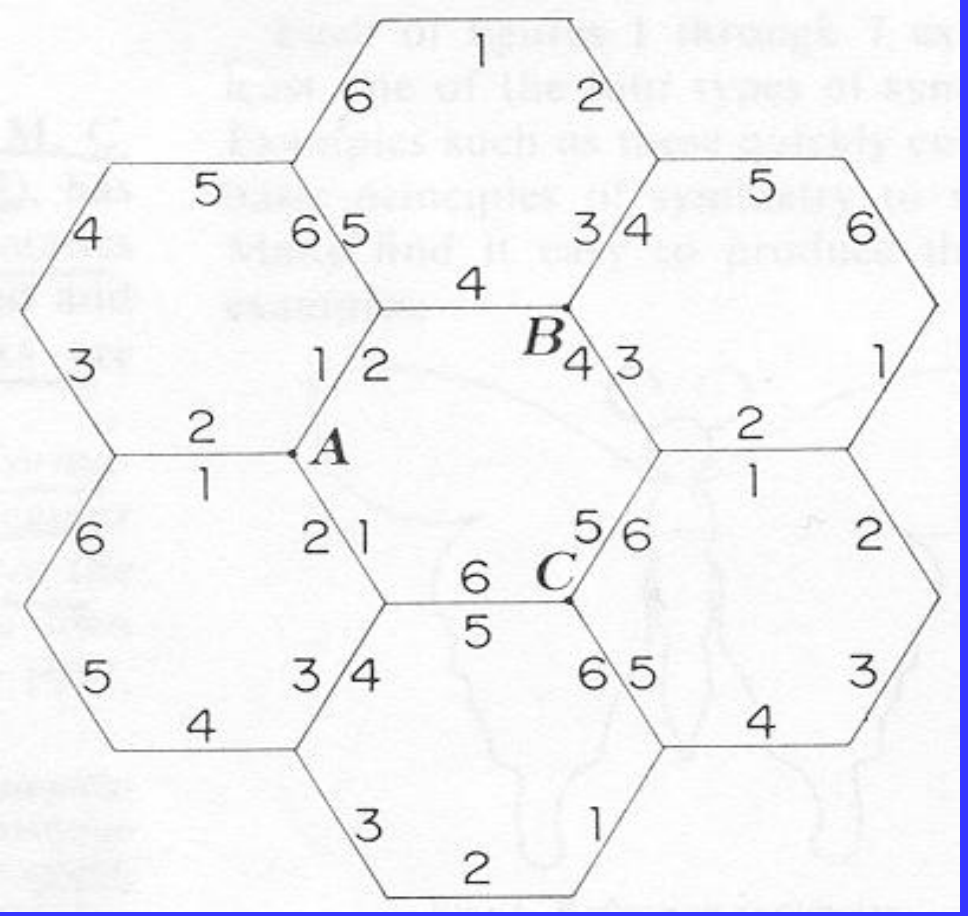



Tile symbol: $a^+b^+c^+d^+$

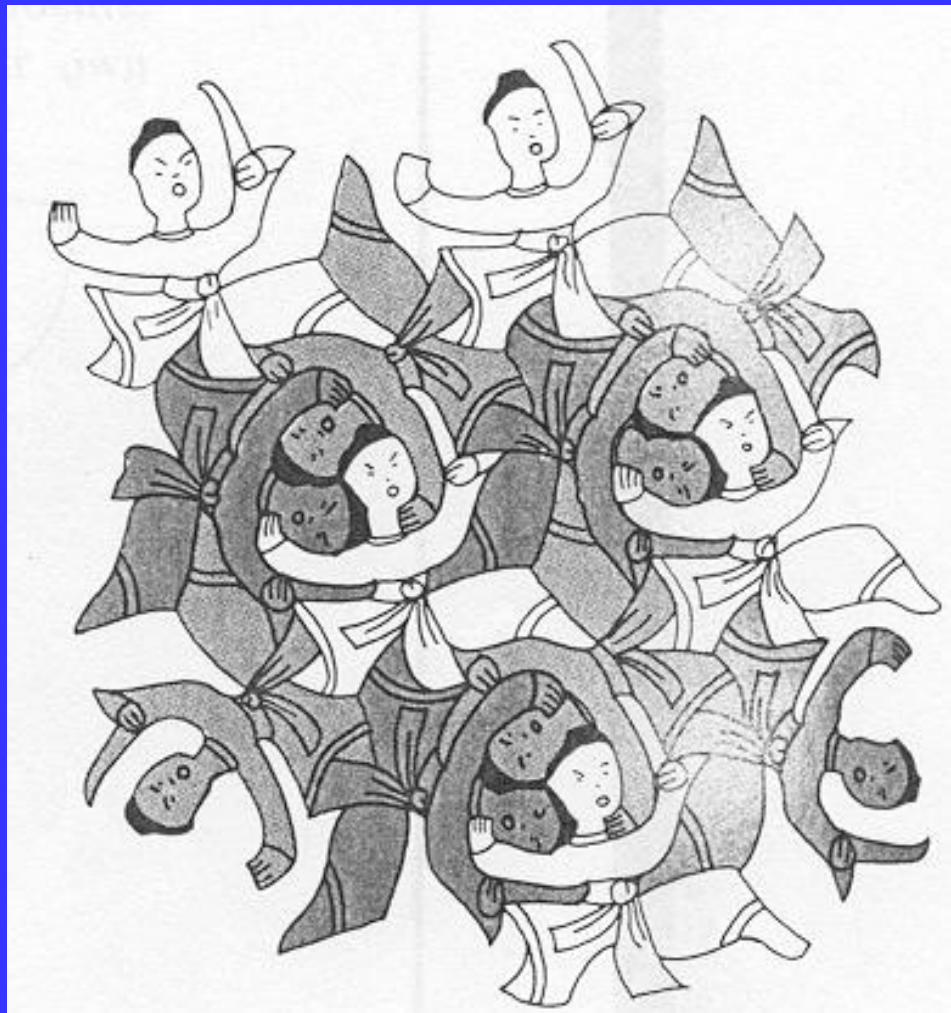
Adjacency symbol: $c^+b^+a^+d^+$



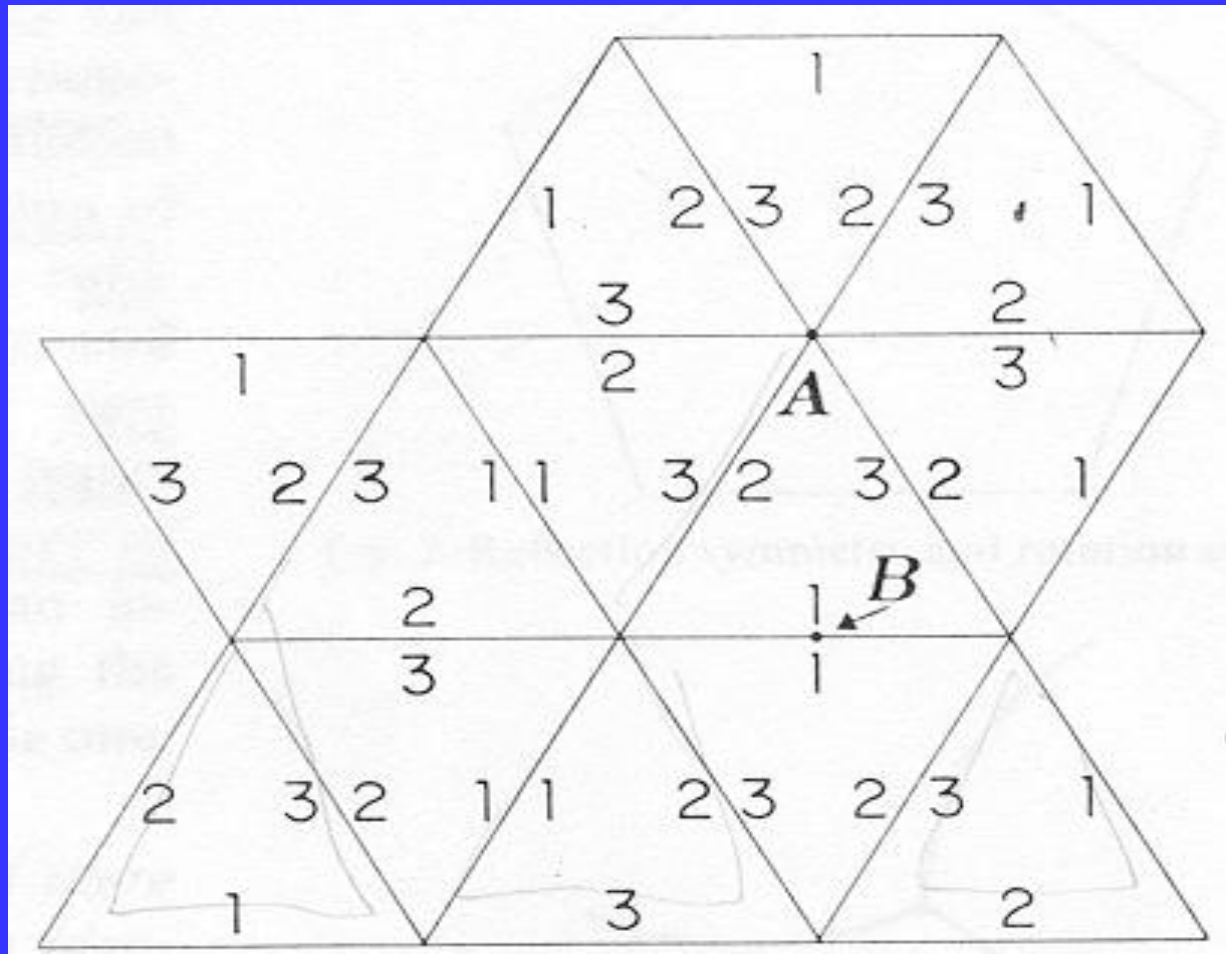


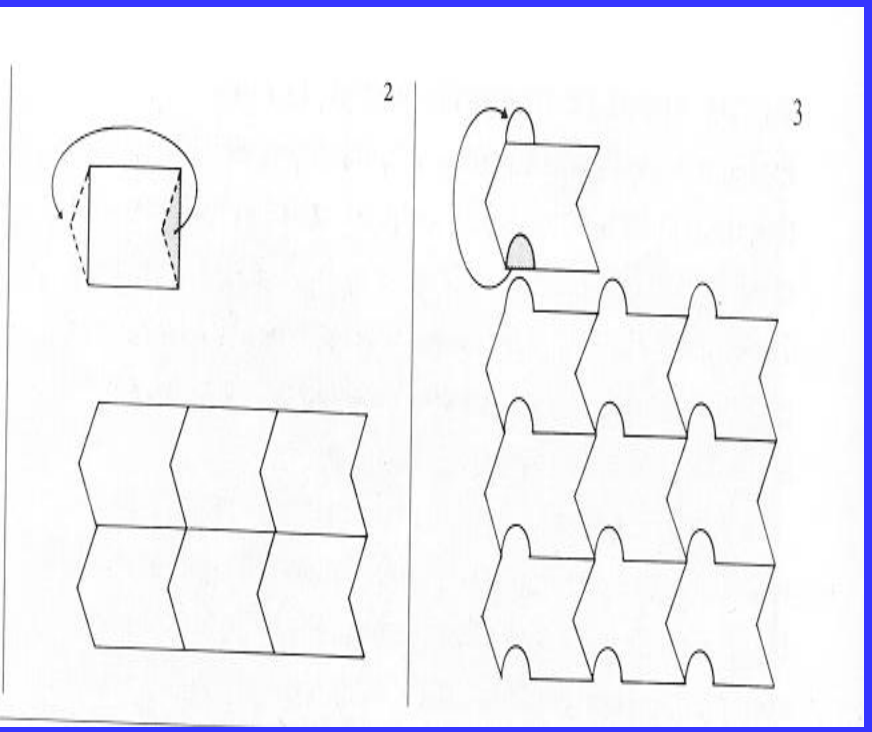



Sides that touch will always fit together:3&4, 5&6,1&2



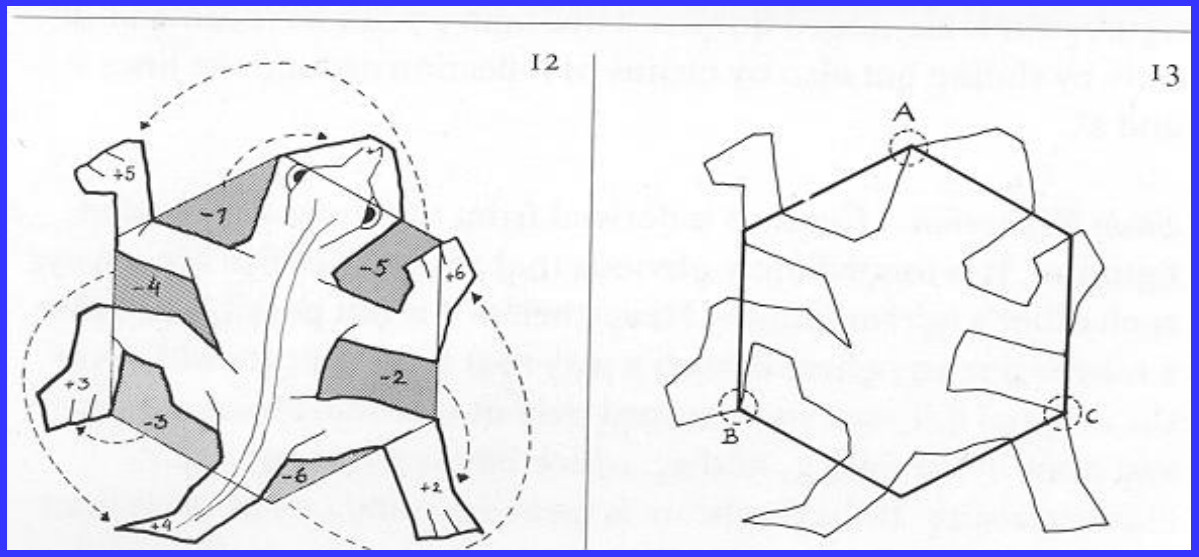
Sample equilateral triangle grid

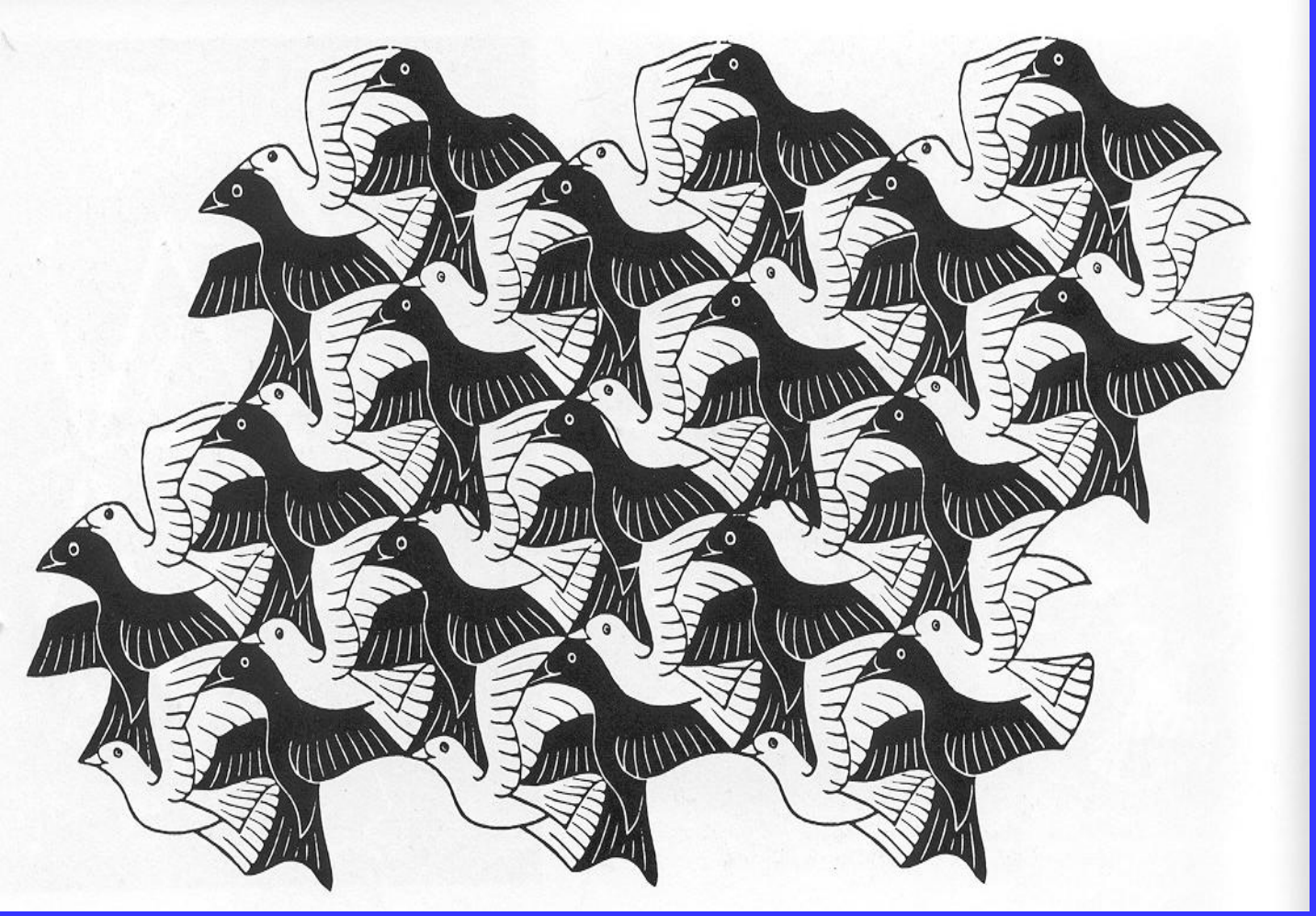





Take off 'space' from one side and slide it over to the other side.

This makes them "fit"



"I want to find happiness in the tiniest things …and I want to…copy these infinitesimally small things as precisely as possible and to be aware of their size."

# *Limits*

"What has been achieved in periodic surface division…? Not infinity…a fragment…If this surface, on which forms fit into one another, were to be of infinite size then an infinite number of them could be shown upon it…beyond the bounds of possibility to fabricate a flat surface stretching endlessly and in all directions."

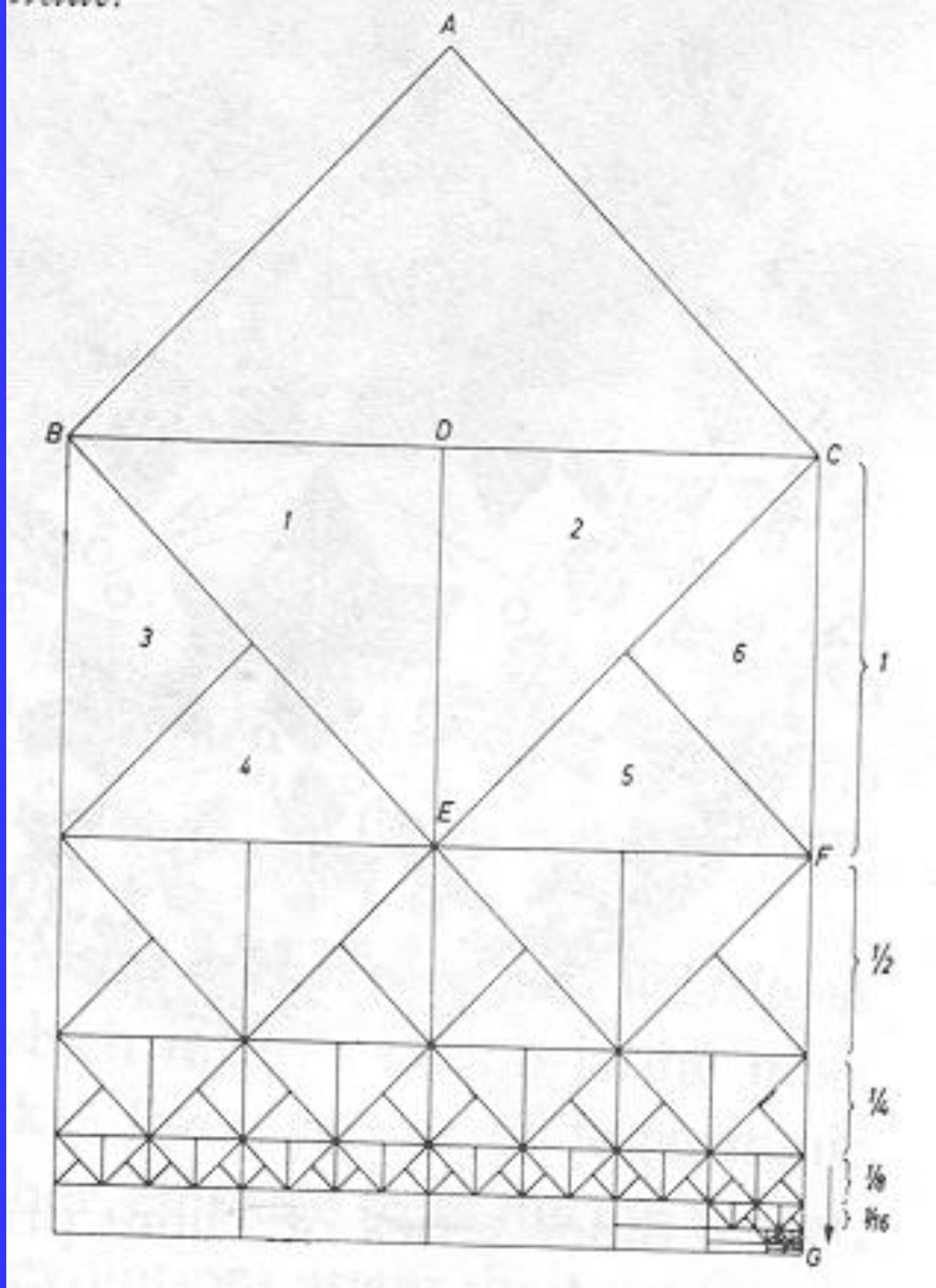
# *Solution*

- 5th symmetry
  - translation
  - rotation
  - reflection
  - glide reflection
  - similarity

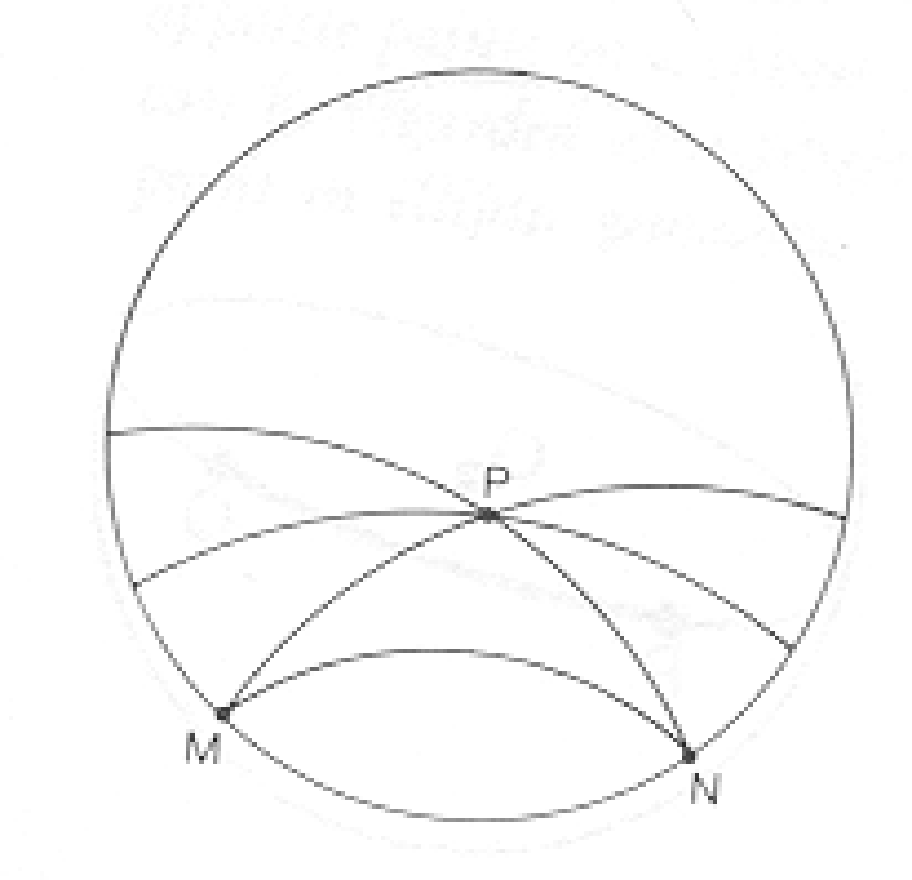
- Relax the requirement of retaining shape and size
- Invariance of shape
- Suggestion of infinity without actually reaching it

# Limit Prints

Grid for *Smaller and Smaller*



- Reduction from without inwards
- Reduction from within outwards
- H.S.M Coxeter
  - Henri Poincare's model



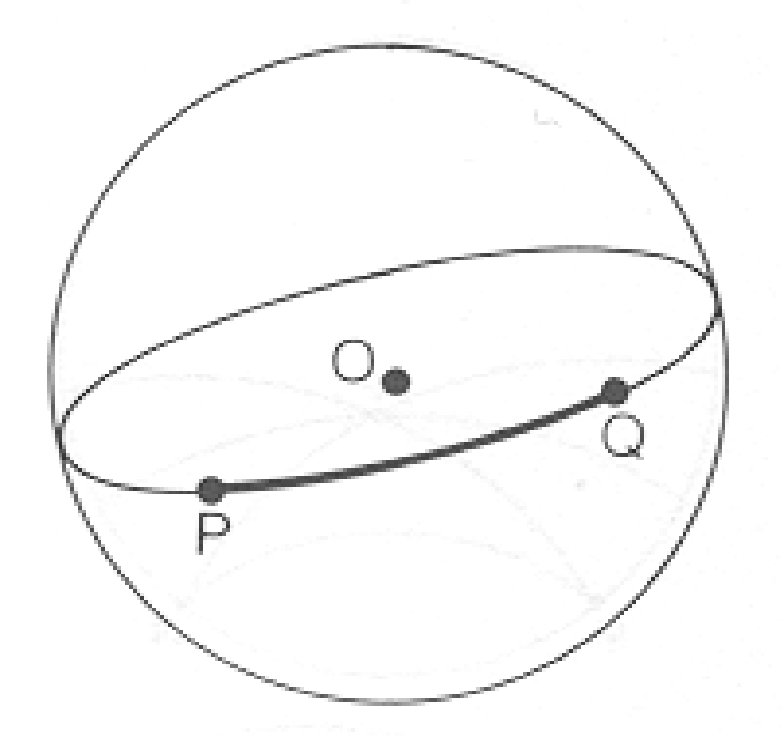
# Euclidean Geometry

- 10 postulates or axioms
  - 5th postulate
    - Playfair's Parallel Postulate

    Given a line l and a point P not on l, there exits one and only one line m, in the plane of P and l, which is parallel to l
    - not evident

# Non- Euclidean Geometry

- Hyperbolic Geometry
  - 3-D sphere
  - lines-great circles



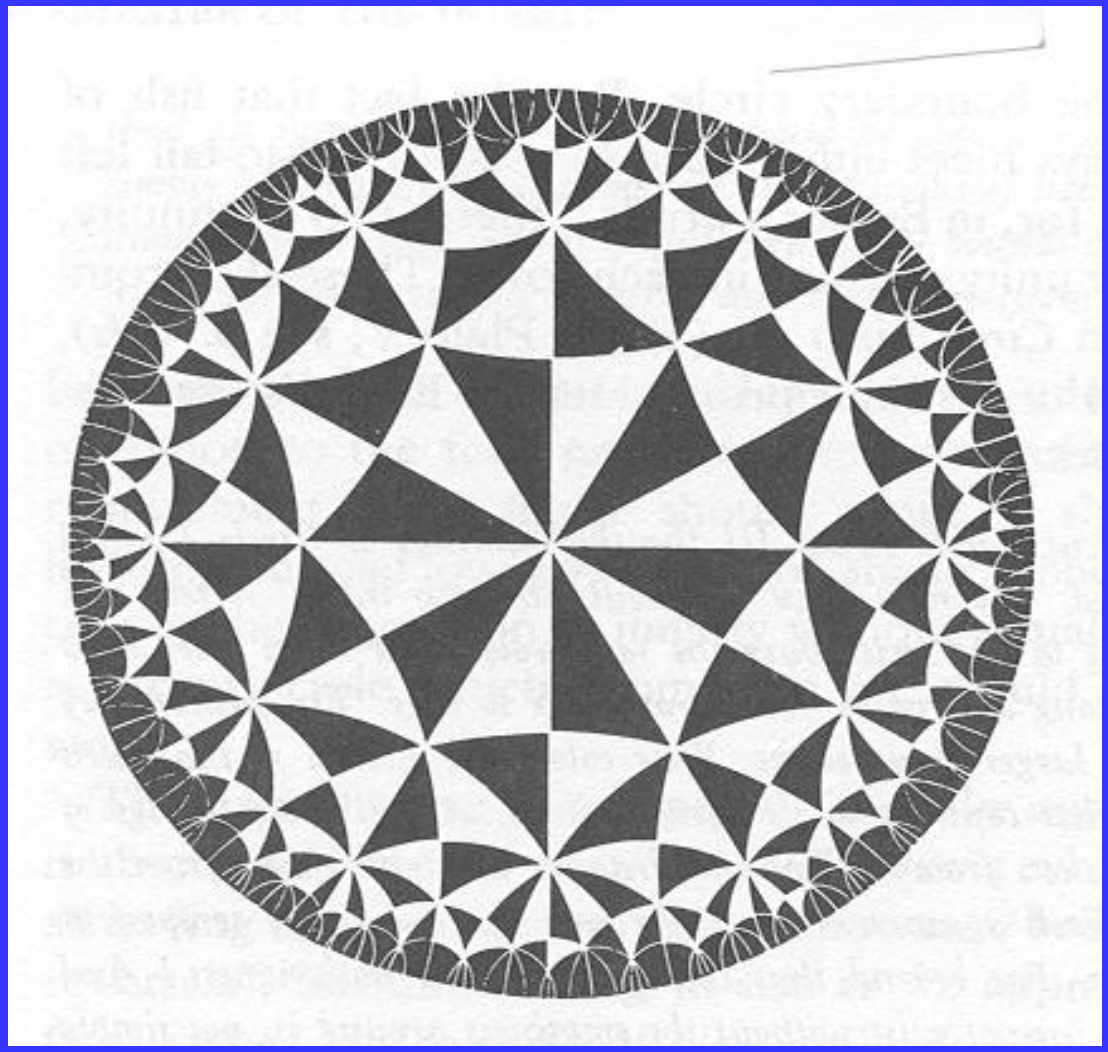
# *Is it possible to draw a parallel to a point outside the line?*

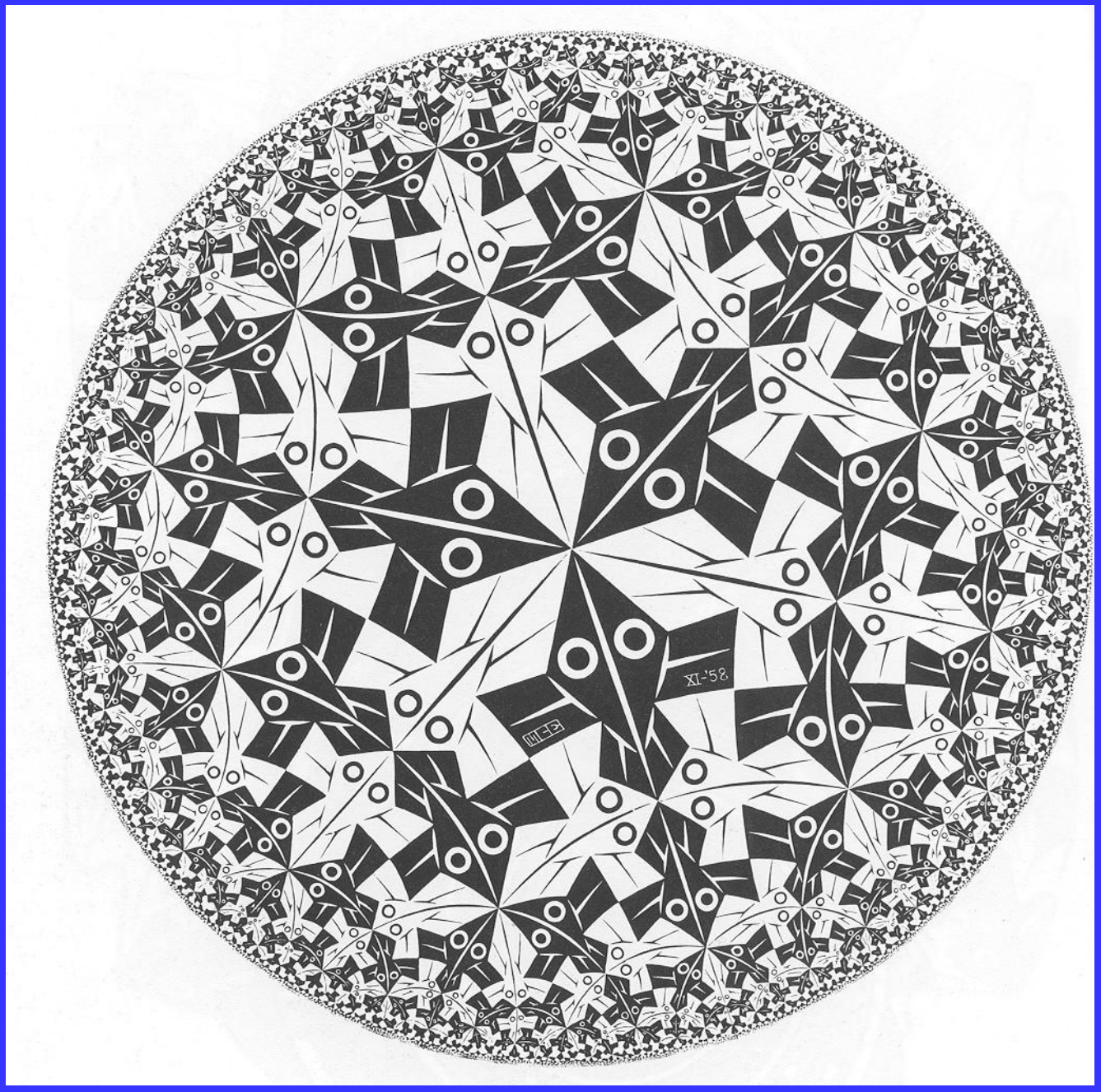
- 2 Great Circles always intersect at 2 diametrically opposite points or antipodes
- Infinitely many straight lines

# *Hyperbolic II Postulate*

- There exists a line l and a point P such that at least 2 distinct lines pass through P that are parallel to l

"I am trying to glean…a method for reducing a plane filling motif …from the center to the edge… His [Coxeter's] hocus-pocus text is of no use to me at all…picture…produce a division of the plane…A circular regular division of the plane, logically bordered on all sides by the infinitesimal, is something truly beautiful…"




XI-'58

More satisfying to Escher with unity of color

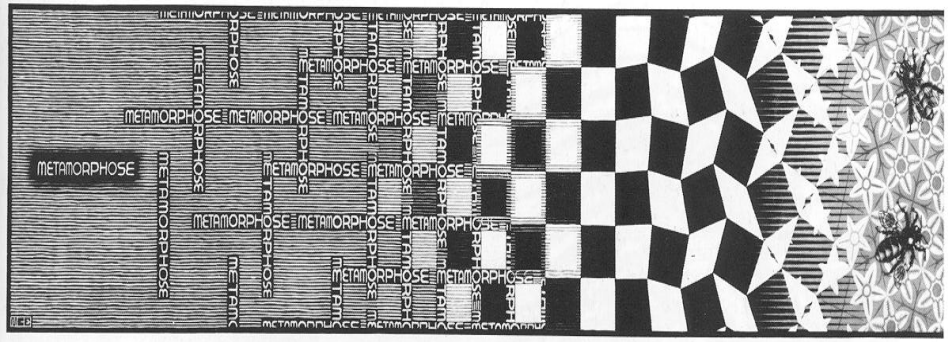
Circle Limit 3

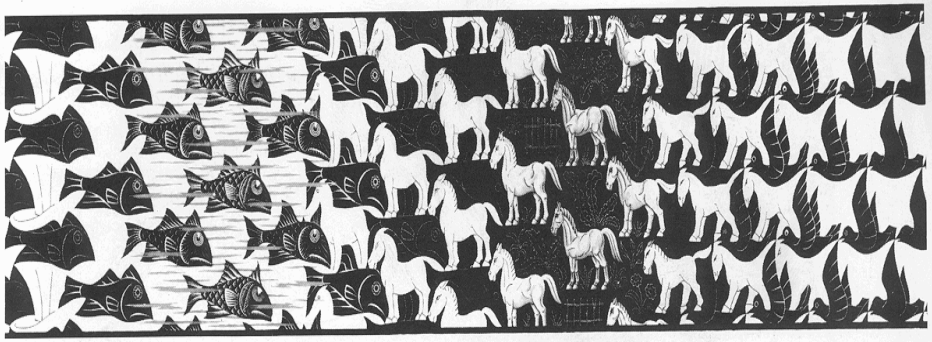
# *Math and Art*

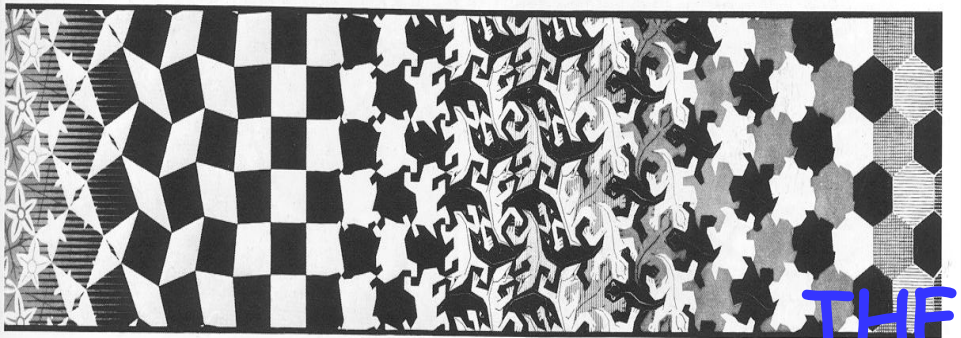
- Math has been invoked in art in an attempt to explain and encourage appreciation for the proportions of volumes of elements in visual composition
- Math is the urge to make a concise intellectual statement about something we have seen in nature

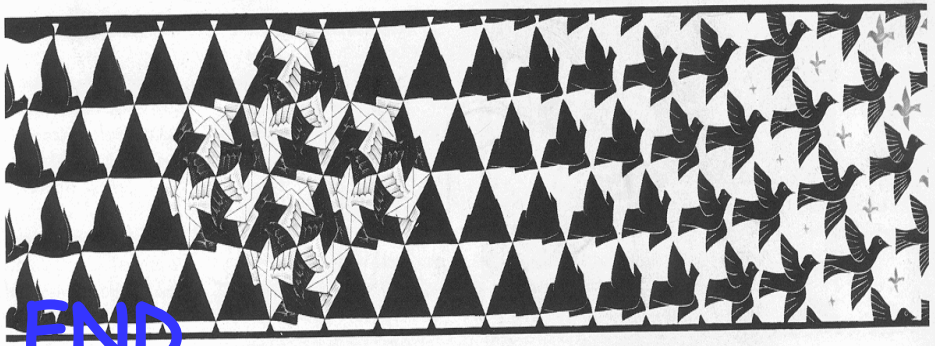
- An attempt by the artist to sensually possess an elegant form that proceeds from abstract logic

- This separates the act of creating art from the science







THE END